SUPPES PREDICATES FOR SPACE-TIME*

ABSTRACT. We formulate Suppes predicates for various kinds of space-time: classical Euclidean, Minkowski’s, and that of General Relativity. Starting with topological properties, these continua are mathematically constructed with the help of a basic algebra of events; this algebra constitutes a kind of mereology, in the sense of Lesniewski. There are several alternative, possible constructions, depending, for instance, on the use of the common field of reals or of a non-Archimedian field (with infinitesimals). Our approach was inspired by the work of Whitehead (1919), though our philosophical stance is completely different from his. The structures obtained are idealized constructs underlying extant, physical space-time.

1. INTRODUCTION

Science is the search for structure. Such a bold claim certainly deserves further elaboration and justification, but it is one which, it has recently been emphasised, has been held by a number of scientists and philosophers. Our aim in this note is two-fold. First, we wish to relate the consideration of such an aim to a long standing programme of work which has sought to develop a mathematically precise treatment of the notion of structure itself.¹ In doing so, we shall analyse a particular illustrative case and thereby generate favourable evidence for the above claim as a whole.

Thus, we shall consider the problem of constructing a set-theoretic structure, in Suppes’s sense², which is capable of providing an axiomatic basis for those notions of space and time which underpin various theories of physics. This problem, which can be located in the context of a long tradition of philosophical reflection about physics, illustrates, we think, in a particularly acute way, the nature of the process of “structuralisation” within contemporary science. In its most general form, it involves the problem of formulating kinds of structures which are sufficiently general and abstract as to capture certain domains of scientific knowledge, such as those modelled by currently accepted theories. In particular, we shall consider a structure which, we believe, captures those space-times which underpin classical mechanics, and the theories of Special and General Relativity. Having said that, we should perhaps make it clear that our intention is not to construct the space-time of physics, but rather to merely

present some mathematical structures which we feel may prove relevant to
the study of space-time in general.

Secondly, we wish to provide further evidence for the claim that, by
means of such structures (conceptual structures modulo a logic) science
attains absolute quasi-truths. The crucial aspect of this thesis rests on the
cumulativist claim that scientific knowledge has tended to stabilize, in the
sense that much of an older theory is retained across theory change by
the new; in particular, what is retained are those aspects of the theory
which refer to the underlying structure. Classical mechanics, for example,
although superseded by relativistic physics, can still be applied, within
determinate limits, to a wide range of domains. In this sense, within these
domains, it remains quasi-true. Analogously, as we shall argue below, a
similar phenomenon can be observed when we consider such structures
for space-time: certain characteristics independently of the theory being
considered, and within determinate domains, remain unaltered through
theory change and in this sense such features can be regarded as "absolute".

In order to argue for this double-headed claim, our paper is divided
into three sections. In the following, we shall outline our philosophical
motivation, as it were. In Section 3, we shall formulate, on the basis
of a determinate algebra of events, the Suppes predicates for the space-
times mentioned above; in particular, the way in which such structures
provide evidence for both the above claims, will be clarified. Finally, as a
conclusion, we shall present a brief exploration of the relationship between
the proposed predicates and our aforementioned philosophical motivation.

2. PHILOSOPHICAL MOTIVATION

As is well known, those concerned with the investigation of the nature
of space-time tend to fall into one of two camps: according to the sub-
stantivalist view, space-time is a metaphysical substance over and above
material things and their (in general polyadic) properties. The alternative
relationist line claims that space-time is nothing but a particular set of
relations which hold between material bodies. It is not our intention to
enter directly into the details of this debate. Two points are worth noting,
however. The first is that the context for current discussion is that of the
"space-time theories" approach: broadly speaking, theories are presented
within a model-theoretical format, involving the specification of so called
"absolute" geometrical objects on the manifold, followed by the specification
of further geometrical objects with dynamical content. Secondly, certain recent analyses have taken a new and potentially interesting direc-
tion. With the further development of the axiomatic method, the problem
has begun to be considered on the basis of an algebra of events and the
formal analysis of certain topological properties of space-time.

However, we should note that such an analysis is concerned with just
some aspects of the space-times involved. To offer an axiomatisation
of them, as we intend to, is to offer a piecemeal approach which models some
basic features only; that is, to provide an implicit characterisation of some
(from our viewpoint) relevant aspects of the subject. By no means do we
claim to have supplied a complete, exhaustive, or detailed analysis of the
issue. Rather, and analogously to the axiomatic approach to the concept of
number for example, only certain features of the space-times discussed are
to be considered.5

3. SUPPES PREDICATES FOR SPACE-TIME

We shall take as our basis ZF set theory with Urelemente (individuals),
which function as objects that are not sets (events, in our case).

Our primitive, central notion is that of an event (which is to be regarded
as individual). Given two events, A and B, there is another event, denoted
by $A \vee B$, which we shall call the junction of A and B, that contains A and
B, as it were. Similarly, given A and B, there is the event $A \wedge B$, called
the meet of A and B, which sums up the common features between them.

Considering two events, A and B, it might be that A is part of B, and
this shall be denoted by $A \leq B$.

Furthermore, we shall postulate the (idealized) existence of a null or
empty event, 0, as well as of a universal event (the universe), 1.

Thus, it seems natural to postulate that the events, satisfying the above
operations and relation, constitute a distributive lattice with greatest and
least elements (1 and 0, respectively). We shall also insist that the set of
events, E, is composed by individuals, not by sets, as in pure set theory.

The above observations together with the point that it seems natural to
suppose the existence of the supremum of an arbitrary family of events
(supremum relative to the junction operation) motivate the following sup-
position: E, with the preceding operations and with 1 and 0, constitutes a
sup-complete distributive lattice with greatest and least elements.6

DEFINITION 1. A lattice such as $\Delta = (E, \wedge, \vee, 0, 1)$ is called a simple
structure of events (SSE).

DEFINITION 2. A family of events, linearly ordered by the relation $\leq$, which
does not have a least element (that is, there is no element of the
family contained in the other ones), is called a normal family of events (NFE).

DEFINITION 3. A lattice such as $\Delta$ in which every event is contained in a NFE is called normal.

Let $\Delta$ be a normal lattice. If $I$ and $J$ are NFE, then $I$ and $J$ are called similar if there is an element of $I$ that contains some element of $J$, and reciprocally. In this case, we shall write: $I \approx J$.

It is easy to verify that $\approx$ is an equivalence relation.

DEFINITION 4. The equivalence classes of $\approx$ are called points. A point $P$ is in event $A$ if there is, in $P$, a NFE $K$ such that $A$ belongs to $K$. The set of points which are in $A$ is represented by $A^*$.

DEFINITION 5. A normal lattice $\Delta$ is called pointed if the following conditions are met:

1. $A^*$ determines $A$.
2. $(A \lor B)^* = A^* \cup B^*$.
3. $(A \land B)^* = A^* \cap B^*$.
4. $0^* = \emptyset$ and $1^* = \text{set } U_\Delta$ of all the points.
5. $(\bigwedge_{i \in I} A_i)^* = \bigcup_{i \in I} (A_i)^*$, for every family $(A_i)_{i \in I}$ of events.

DEFINITION 6. Let $\Delta$ be a pointed lattice. We shall denote by $\tilde{A}_\Delta$ the class of all the sets $X$ such that $X = A^*$, for some event $A$ of $\Delta$.

It is easy to prove the following theorem.

THEOREM. If $\Delta$ is a normal, pointed lattice, then $\tilde{A}_\Delta$ is a topology; that is, $E_\Delta = (U_\Delta, A_\Delta)$ is a topological space.

The preceding considerations motivate the following definition:

DEFINITION 7. A space-time simple structure (STSS) is a normal, pointed lattice. The elements of $\tilde{A}_\Delta$ are the open sets of space-time and the elements $U_\Delta$ are the points (or instant-points) of space-time.
It is plain that a space-time simple structure captures crucial aspects of our intuitions regarding physical space-time, when \( E_\Delta \), for instance, is supposed to be a connected, simply connected space, etc. However, we shall not enter into the details of this topic. At present, our intention is only to show how the underlying space-time of classical rational mechanics as well as those of Special and General Relativity can be obtained. Our exposition is by no means aimed towards a logically and mathematically detailed analysis of these structures. The reader with some acquaintance with these topics will be able to proceed in a more rigorous way.

The main point consists in the claim that any axiomatization of space-time should include the notion of STSS, which sums up the chief topological features of space-time.

The space-time of classical mechanics can then be defined by the following structure.

DEFINITION 8. A classical space-time structure is a STSS whose associated topological space, in an obvious sense, is homeomorphic to \( \mathbb{R}^4 \).

The metric and related features of \( \mathbb{R}^4 \) are then induced to STSS by the homeomorphism.\(^9\)

In similar fashion, we might conceive of the space-time of Special Relativity (homeomorphic to a Minkowski manifold) as well as that of General Relativity (homeomorphic to a convenient Riemannian differentiable manifold).

Thus, we obtain set-theoretic structures corresponding to the various kinds of space-time used within physics (which offers some support for the first claim stated in the introduction to this paper).

Furthermore, the structures obtained make it plain that, despite certain idealizations and variations, the basic space-time presents some features that can not be withdrawn. There are some absolute aspects, as it were, regarding space-time (see our introduction's second claim). In a first approach (within classical mechanics as well as in Relativity theory), for instance, the space-time is continuous.

4. PHILOSOPHICAL REMARKS

In his introductory text, Sklar credits the substantivalist with believing that space-time is a structure over and above material things and their properties (Sklar 1992). This is clearly not enough to distinguish it from the relationist view. If by "properties" is meant monadic properties only, a relationist could surely be said to share this belief. A Leibnizian relationist, however, would
not, where a Leibnizian relationist is taken to be a relationist who holds that all relational properties can be reduced to monadic ones. The issue of the nature of the relata then looms large: to what monadic properties of physical objects can spatio-temporal relations be reduced? It is precisely a failure to come up with a satisfactory answer to this question that propels the Leibnizian relationist to the position that physical objects cannot be the ultimate relata and that only non-spatio-temporal entities such as monads can fulfill this role. (As is well known, it is a matter of debate whether, in fact, Leibniz himself believed that such a reduction could be effected.) A substantivalist, of course, is going to look on this as something approaching a reductio but the claim that spatio-temporal relations are non-supervenient upon the monadic properties of physical objects in this sense, rather than being further grist to her mill, represents a dramatic departure from the traditional view she would typically be associated with. (If physical objects are regarded as nothing more than bundles of properties themselves then it could be suggested that spatio-temporal relations are only weakly non-supervenient on these properties in the sense that they are not determined by them but they are—epistemically at least—dependent on such properties. The problem with this line is its reliance on the Principle of the Identity of Indiscernibles to guarantee individuation, and this is a Principle which has come under much criticism in recent years.)

Returning to Sklar’s comment, it does mesh well with a recent attempt to break out of a related dialectic between realism and anti-realism (epistemology and ontology are here further entwined). On the one hand, we have the ‘no miracles’ argument which asserts that from any perspective other than a realist one, the success of science—particularly with regard to theory unification— is a miracle. On the other, there is the ‘changing furniture’ argument, which highlights the difficulties in maintaining a realist position in the face of theory change and the concomitant changes in what is taken to constitute the ‘furniture of the world’.

According to “structural realism” the concerns behind these two arguments, pulling, as they do, in different directions, can be assuaged once it is accepted that the ontological referent of theories and that which provides continuity across theory change, is some kind of structure. We say “some kind” because here opinions differ, with some arguing that the structure is essentially mathematical, others that it is phenomenological. The programme has yet to be developed in full detail although certain prominent features can be discerned as it is applied to various fields.

One of these might be the field of space-time theories. Thus, pursuing the ontic aspect first of all, one might argue that space-time is not a substance, but a structure which exists over and above material things and
their (non-spatio-temporal) properties. Perhaps the most well known exponent of such a view in this context is Stein who, in a famous debate with Grünbaum, characterised the difference in their ontological perspectives thus: 'You tend to think of the world in terms of "things" — "primary substances" in Aristotle's sense. I do not: I tend, rather, to think (Platonically?) of "structures" and "aspects of structure" ("Forms"?)' (Stein 1977, 395). The results set down above may then be regarded as delimiting the nature of this structure.

Turning to the epistemic issue, we can, as we noted above, identify the element of continuity over theory change through these results. Simply put, this element comprises the space-time simple structure.

There is therefore a concordance between the ontic and epistemic aspects: space-time theories, like all theories in general, can be said to be partially or quasi-true, in the formal sense spelled out elsewhere. Of course, it would be absurd to say that the theoretical entities postulated by such theories partially exist. Rather, there is a common underlying structure — spatio-temporal in this context — about which we come to know more and more. The appropriate representation of this structure, we believe, is in set-theoretic terms.

NOTES

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1 This is a problem which has preoccupied one of the authors (Newton da Costa) for a long time. A preliminary version of his ideas in this regard, in joint work with Rolando Chuaqui, was presented in da Costa and Chuaqui (1988). Recently, however, a simpler characterisation was obtained (retaining Bourbaki's proposals as their main basis). We intend to present, in later papers, some applications of these ideas in the context of specific scientific theories (from physics as well as from mathematics), and also to relate them to some theses defended within the structuralist programme.

2 For a clarification of this concept, see for instance, Suppes (1957; 1967 and 1988).

3 The concepts of quasi-truth and, in close connection with this, of partial structures have received considerable attention during the last few years. A formally rigorous and detailed approach can be found, for instance, in Mikenberg et al. (1986), as well as in da Costa and Chuaqui (1994); a general exposition of the theory, and also some of its philosophical motivations are presented in da Costa (1989); finally, some possible applications within the philosophy of science's domain are explored in da Costa and French (1990); cf. also da Costa and French (1993).

4 For a recent discussion, see Earman (1989).

5 An interesting approach to this problem, in similar lines to the ones adopted here, can be found in Carnap (1958, 197–210); cf. also Whitehead (1919) and Russell (1914).

6 For a detailed study of the geometrical aspects of lattice theory, see Blumenthal and Menger (1970).
7 For the definition of a 'normal family of events' see Menger (1940).
8 Likewise for the definition of 'similarity' between NFE's.
9 To be absolutely explicit here: when we say that there is a homeomorphism between our structure and $\mathbb{R}^4$, we implicitly suppose that the metric etc. of $\mathbb{R}^4$ is transferred to our structure; and likewise for Minkowski; "Riemannian", etc. space-times.
10 For some recent, interesting discussions, see Worrall (1989), Chiappini (1989), Torretti (1990), Saunders (1993) and Ladyman (1998). It is to Ladyman that we owe the clear expression of this point.

REFERENCES


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