Truth, Quasi-Truth and Paraconsistency

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1. Introduction

The notion of truth plays, in one form or another, a prominent role in several domains of inquiry: from semantics to the philosophy of language, through the philosophy of science and logic. Crucial concepts, such as model and logical consequence, as well as accounts of methodological and epistemological notions, such as confirmation and belief change, are articulated in terms of truth.

In all of these applications, the notion of truth has not remained unchanged. As is well known, there are several distinct theories of truth: the theories of correspondence, the coherence theory, as well as the pragmatist and deflationary accounts. Each of these theories stresses different aspects of truth, and they are not entirely compatible. This may suggest that truth is an inconsistent notion. But is this really so?

In this paper, I want to discuss the relationship between inconsistency and truth. And I shall do that in a double-headed way. Firstly, in section 1, I shall examine some arguments for and against the idea that truth is indeed inconsistent. Such arguments, as we shall see, are mostly inconclusive, and because of this it seems to me that an agnostic attitude towards the nature of truth (vis-à-vis its inconsistency) is advisable. Having said that, I do think that those who defend the inconsistency of truth still have a case. If nothing else, most arguments against their views are clearly question-begging. So, without assuming anything stronger than an agnosticism, my second aim is to provide a formal framework to explore inconsistent formulations of the notion of truth (which will be done in section 3). Since this framework is articulated in terms of the resources provided by Newton da Costa’s and Steven French’s partial structures approach, I shall first spell out, in section 2, the main components of this view. Finally, in section 4, an application of this framework will be provided, by critically discussing some aspects of Graham Priest’s dialetheist proposal to the effect that there are true contradictions.

As we shall see, this formal framework can be taken by those who actually believe in the inconsistency of truth as a further setting for the exploration of their ideas. But the unbeliever may gain some comfort by noticing how sober this setting is. In this regard, the (possible) development of an inconsistent

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1For a discussion of certain versions of these proposals, see Kirkham [1992].

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approach to truth may not be so weird as it may initially seem — or so I shall argue.

2. Truth and inconsistency

Is truth inconsistent? Someone may claim that the answer is negative. It is not truth, so the argument goes, but its theories which are inconsistent, and thus if there are any difficulties here, they should be traced back to the latter, not to the former. (Let us call this the consistentist view.) As opposed to this, two arguments can be presented. Firstly, since these are theories of truth, it may well be that truth is inconsistent, given that it led to these theories. This argument, however, is not conclusive, since it may be claimed that truth is in fact consistent, but it underdetermines its theories just as, in scientific practice, mutatis mutandis, empirical data underdetermine scientific theories. The problem with this reply is that the relationship between truth and its theories is not analogous to that between empirical data and scientific theories. A crucial requirement for scientific theories is to provide new predictions of empirical phenomena. However, we do not require of theories of truth to supply new predictions about truth. Indeed, it is not even clear how these predictions would look like. What we do require of theories of truth are answers to two questions: (1) How is truth to be understood? (2) What is the point of introducing a truth notion? This already indicates that we are concerned with interpretation issues.

Given these two questions, someone may then argue that the relationship between truth and its theories is closer to the relationship between a scientific theory and its interpretations than to empirical data and certain scientific theories. In other words, we ought to move one level up from empirical data to find the appropriate analogy between these notions. Indeed, an interpretation of a scientific theory is usually thought of as providing answers quite close to (1) and (2), namely: (1') How is a scientific theory to be understood? (2') What is the point of introducing such a theory? Of course, in each case, the notions of understanding and the point of formulating the theory in question are different, since in one case we are dealing with a scientific theory, in the other with the notion of truth. Nonetheless, they have something in common: in both cases one is concerned with an interpretative task, whether we are examining truth or empirical theories. The moral to be drawn is then clear: from the fact that there are inconsistent interpretations of a scientific theory, we cannot conclude that the theory itself is inconsistent. At best, it leaves some parameters open to (non-empirical) examination. Similarly, we cannot conclude that truth is inconsistent from the fact that there are inconsistent theories of it.

The point is well taken, but it is not enough to settle the matter. The difficulty with this argument derives from an asymmetry between the two sets of questions just considered. If we are asked to spell out the content of question (1'), which involves (a request for) the understanding of a scientific theory, we would obtain something along these lines: What is the world picture provided by such a theory if the latter is true?\(^2\) Similarly, as for (2'), the point of

\(^2\)Van Fraassen's own perspicuous way of spelling out an interpretative question in science
formulating a particular scientific theory, at least in a realist interpretation of science, is the discovery of the truth (or at least the approximate truth) about the domain investigated by this theory. The same holds for an empiricist interpretation, such as van Fraassen's constructive empiricism (see his [1980], [1985] and [1989]). According to this proposal, truth is not taken as the aim of science, but something weaker is, namely empirical adequacy (see van Fraassen [1980], p. 12 and p. 64). The latter is then spelled out as truth with regard to observable phenomena. Thus, the content of interpretative questions about science involves, both in realism and in empiricism, the notion of truth. In this respect, answers to questions (1) and (2), which concerns interpretative issues about truth itself, are presupposed when we answer questions (1') and (2'). As a result, there is a crucial distinction between these kinds of question, and the analogy between them cannot be used so straightforwardly.

The second argument against the consistentist suggestion that it is not truth, but its theories which are inconsistent, is as follows. This suggestion faces considerable difficulties in reconciling its account of truth as a consistent notion with the undeniable fact that there are several incompatible theories of truth (this is, of course, the upshot of the previous arguments). Thus, the consistentist ought to establish that despite not being necessarily inconsistent, truth may lead to inconsistencies, paradoxes and incompatible theories. However, (i) is this a coherent suggestion? And if so, (ii) how might one distinguish it from the consistentist proposal we have just considered in the previous paragraphs? As for (i), it may be denied that this suggestion is coherent, since one way of saying that a theory T is inconsistent is, of course, to derive a 'contradiction' from T. And apparently, the thought here is that we may distinguish between truth being inconsistent and the possibility of deriving a 'contradiction' from the truth (whatever this means). This leads us directly to (ii). The first consistentist proposal assumes that truth is consistent, whereas the second adopts a weaker, 'modal' version to the effect that it is possible that truth is inconsistent, and then go on to distinguish this possibility from the possibility of deriving a 'contradiction' from the truth. So, the distinction between the second version and the first collapses if there is no way of demarcating between the property of being inconsistent and of deriving a 'contradiction' from the truth. And there may be ways of accomplishing that, perhaps by exploring an appropriately non-classical (paraconsistent?) model theory. What is not clear is on what grounds this non-classical approach would be open for a consistentist. If one is trying to defend the claim that truth is not inconsistent, why should a paraconsistent strategy be of any help? (One of the tasks of the present paper is to provide

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In this sense, constructive empiricism does not reject, but only restrict, the use of the notion of truth in interpreting science. Although truth is not (or does not need to be taken as) the aim of science, it is not abandoned, since empirically adequate theories are true if we only consider the observable phenomena.
a framework in which this and related issues can be properly discussed and evaluated.

This second argument touches upon a delicate point: the 'strength' of truth, the way in which truth spells out the relationship between language and reality, requiring a tidy relationship between them. Under this construal, a sentence is true if it describes appropriately, and in every detail, the 'state of affairs' that obtains in the world. (This is obviously closer to the correspondence account of truth.) If this is so, we have here a quite strong notion, since it demands a close connection between reality and our ways of representing it. This is the ontological component of truth. But there is a further component which also accounts for the strength of this notion, and it is properly semantic. It depends, as it were, on the expressiveness of truth. Several semantic paradoxes, the Liar being the obvious example, depends upon the notion of truth (or notions closely related to it). (Of course, they also depend on the expressive resources of the language under consideration — for instance, whether it is or not semantically closed. But notice that this is a feature that has been discovered and studied in order to control the paradoxes.) So, the fact that truth leads to the formulation of several paradoxes clearly suggests the strength of this notion.4

The arguments discussed thus far do not seem to be conclusive either in establishing that truth is inconsistent, or in establishing its denial. This suggests that perhaps the best attitude towards the question of whether truth is inconsistent is that of agnosticism. Thus one avoids being committed to a claim about the nature of truth. Despite avoiding this commitment, it is still possible to provide a framework in which issues about the relationship between truth and inconsistency can be addressed. In what follows, by using the formal machinery provided by da Costa's and French's partial structures approach, I wish to elaborate on this suggestion. The idea is to explore some models to represent inconsistent and incomplete information, which if not true is, at least, partially true. The results which can be obtained in terms of this framework will then provide further grist to the agnostic's mill — or so I shall argue.

3. Partial structures and quasi-truth

As articulated by da Costa and French, the partial structures approach relies on three main notions: partial relation, partial structure and quasi-truth.5 One of the main motivations for introducing this proposal comes from the need for supplying a formal framework in which the 'openness' and 'incompleteness' of information dealt with in scientific practice can be accommodated in a unified way (see da Costa and French [1998]). This is accomplished by extending, on the one hand, the usual notion of structure — in order to model the partialness

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4 According to Priest, the best arguments for the inconsistency of truth are the semantic paradoxes, in particular, the liar (see Priest [1987], pp. 11-34). I shall consider Priest's view in section 4, below.

5 This proposal was first presented in Mikenberg, da Costa and Chuaqui [1986], and da Costa [1986a]. It was then extended in da Costa and French [1989], [1990], [1993a], [1993b], [1995] and [1998], French [1997], Bueno [1997], Bueno [1998], Bueno and de Souza [1996], French and Ladyman [1999] and [1997].
of information we have about a certain domain (introducing then the notion of a partial structure) —, and on the other hand, by generalising the Tarskian characterisation of the concept of truth for such ‘partial’ contexts (advancing the corresponding concept of quasi-truth).

The first step then, in order to introduce a partial structure, is to formulate an appropriate notion of partial relation. When investigating a certain domain of knowledge \( \Delta \), we formulate a conceptual framework which helps us in systematising and organising the information we obtain about \( \Delta \). This domain is then tentatively represented by a set \( D \) of objects, and is studied by the examination of the relations holding among \( D \)'s elements. However, we often face the situation in which, given a certain relation \( R \) defined over \( D \), we do not know whether all the objects of \( D \) (or \( n \)-tuples thereof) are related by \( R \). This is part and parcel of the ‘incompleteness’ of our information about \( \Delta \), and is formally accommodated by the concept of partial relation. The latter can be characterised as follows. Let \( D \) be a non-empty set. An \( n \)-place partial relation \( R \) over \( D \) is a triple \( \langle R_1, R_2, R_3 \rangle \), where \( R_1 \), \( R_2 \), and \( R_3 \) are mutually disjoint sets, with \( R_1 \cup R_2 \cup R_3 = D^n \), and such that: \( R_1 \) is the set of \( n \)-tuples that belong to \( R \), \( R_2 \) is the set of \( n \)-tuples that do not belong to \( R \), and \( R_3 \) is the set of \( n \)-tuples for which it is not defined whether they belong or not to \( R \). (Notice that if \( R_3 \) is empty, \( R \) is a usual \( n \)-place relation which can be identified with \( R_1 \).)

However, in order to represent appropriately the information about the domain under consideration, we need of course a notion of structure. The following characterisation, spelled out in terms of partial relations and based on the standard concept of structure, is meant to supply a notion which is broad enough to accommodate the partiality usually found in scientific practice. The main work is, of course, done by the partial relations. A partial structure \( S \) is an ordered pair \( \langle D, (R_i)_{i \in I} \rangle \), where \( D \) is a non-empty set, and \( (R_i)_{i \in I} \) is a family of partial relations defined over \( D \).

It should be pointed out that the fact that partial relations and partial structures are partial is due to the ‘incompleteness’ of our knowledge about the domain under investigation — with further information about this domain, a partial relation may become total. Thus, the partialness modelled by the partial structures approach is not understood as an intrinsic, ontological ‘partialness’ in the world — an aspect about which the empiricist will be glad to remain agnostic. We are concerned here with an ‘epistemic’, not an ‘ontological’ partialness.

We have now defined two of three basic notions of the partial structures approach. In order to spell out the last, and crucial one — namely, quasi-truth —, we will need an auxiliary notion. The idea is to use, in the characterisation of quasi-truth, the resources supplied by Tarski's definition of truth.\(^6\) However,
since the latter is only defined for full structures, we have to introduce an intermediary notion of structure to link it to the former. And this is the first role of those structures which extend a partial structure \( A \) into a full, total structure (which are called \( A \)-normal structures). Their second role is purely model-theoretic, namely to put forward an interpretation of a given language and, in terms of it, to characterise basic semantic notions. The question then is: how are \( A \)-normal structures to be defined? Here is one answer. Let \( A = \langle D, R_i \rangle_{i \in I} \) be a partial structure. We say that the structure \( B = \langle D', R'_i \rangle_{i \in I} \) is an \( A \)-normal structure if (i) \( D = D' \), (ii) every constant of the language in question is interpreted by the same object both in \( A \) and in \( B \), and (iii) \( R'_i \) extends the corresponding relation \( R_i \) (in the sense that, each \( R'_i \), supposed of arity \( n \), is defined for all \( n \)-tuples of elements of \( D' \).

However, given a partial structure \( A \), there might be too many \( A \)-normal structures. We need to provide constraints to restrict the acceptable extensions of \( A \). In order to do that, we need a further auxiliary notion (see Mikenberg, da Costa and Chuqui [1986]). A pragmatic structure is a partial structure to which a third component has been added: a set of accepted sentences \( P \), which represents the accepted information about the structure’s domain. (Depending on the interpretation of science which is adopted, different kinds of sentences are to be introduced in \( P \): realists will typically include laws and theories, whereas empiricists will add mainly certain laws and observational statements about the domain in question.) A pragmatic structure is then a triple \( A = \langle D, R_i, P \rangle_{i \in I} \), where \( D \) is a non-empty set, \( (R_i)_{i \in I} \) is a family of partial relations defined over \( D \), and \( P \) is a set of accepted sentences. The idea is that \( P \) introduces constraints on the ways that a partial structure can be extended.

Our problem now is, given a pragmatic structure \( A \), what are the necessary and sufficient conditions for the existence of \( A \)-normal structures? We can now spell out one of these conditions (see Mikenberg, da Costa and Chuqui [1986]). Let \( A = \langle D, R_i, P \rangle_{i \in I} \) be a pragmatic structure. For each partial relation \( R_i \), we construct a set \( M_i \) of atomic sentences and negations of atomic sentences, such that the former correspond to the \( n \)-tuples which satisfy \( R_i \), and the latter to those \( n \)-tuples which do not satisfy \( R_i \). Let \( M = \cup_{i \in I} M_i \). Therefore, a pragmatic structure \( A \) admits an \( A \)-normal structure if, and only if, the set \( M \cup P \) is consistent.

Assuming that such conditions are met, we can now formulate the concept of quasi-truth. A sentence \( \alpha \) is quasi-true in \( A \) according to \( B \) if (i) \( A = \langle D, R_i, P \rangle_{i \in I} \) is a pragmatic structure, (ii) \( B = \langle D', R'_i \rangle_{i \in I} \) is an \( A \)-normal structure, and (iii) \( \alpha \) is true in \( B \) (in the Tarskian sense). If \( \alpha \) is not quasi-true in \( A \) according to \( B \), we say that \( \alpha \) is quasi-false (in \( A \) according to \( B \)). Moreover, we say that a sentence \( \alpha \) is quasi-true if there is a partial structure \( A \) and a corresponding \( A \)-normal structure \( B \) such that \( \alpha \) is true in \( B \) (according to Tarski’s account). Otherwise, \( \alpha \) is quasi-false.

The idea, intuitively speaking, is that a quasi-true sentence \( \alpha \) does not nec-
essarily describe, in an appropriate way, the whole domain to which it refers, but only an aspect of it — the one modelled by the relevant partial structure \( A \). After all, there are several different ways in which \( A \) can be extended to a full structure, and in some of these extensions \( \alpha \) may not be true. As a result, the notion of quasi-truth is strictly weaker than truth: although every true sentence is (trivially) quasi-true, a quasi-true sentence is not necessarily true (since it may be false in certain extensions of \( A \)).\(^7\) As we shall see, this is also an important feature of this notion.

4. Quasi-truth and inconsistency

Let us suppose that we want to develop an inconsistent account of truth. The motivation for doing this may come from inconsistencies found in science, from the semantic paradoxes or from the strong features of the notion of truth. (Notice that the development of this account is entirely compatible with an agnosticism about the nature of truth, given that we are only exploring a possible ‘model’ according to which truth may behave.) In order to achieve this task, two strategies can be articulated, both of them resulting from the partial structures approach.

The first, which has already been explored in da Costa’s and French’s works, is to notice that a sentence \( \alpha \) and its negation \( \neg \alpha \) can both be quasi-true. This is due to the fact that, depending on the way in which a partial structure \( A \) has been extended to a full, \( A \)-normal structure \( B \), a sentence \( \alpha \) can be quasi-true (in \( A \) according to \( B \)), but also, since the extensions supplied by \( A \)-normal structures are not unique, there might be a different \( A \)-normal structure, say \( B' \), in which \( \neg \alpha \) is quasi-true. Therefore, if we denote the quasi-truth operator by \( Q_T \) (when applied to a sentence \( \alpha \), it is to be read: ‘\( \alpha \) is quasi-true’) we have that

\[
(1) \quad Q_T(\alpha) \land Q_T(\neg \alpha).
\]

The question to be raised at this level is whether (1) implies

\[
(2) \quad Q_T(\alpha \land \neg \alpha).
\]

Of course, if we are to devise an inconsistent truth notion, we may require that (1) and (2) are at least materially equivalent. The answer to this question depends on the formal features of the particular setting one considers. In the present formulation of quasi-truth, which is based on classical logic (all \( A \)-normal structures are classical), a contradiction of the form \( (\alpha \land \neg \alpha) \) is not quasi-true, even if \( Q_T(\alpha) \) and \( Q_T(\neg \alpha) \), since there is no \( A \)-normal structure which is a model of \( (\alpha \land \neg \alpha) \). Thus, (1) does not imply (2). This happens because the existence of \( A \)-normal structures, in terms of which the quasi-truth of \( \alpha \) is defined, depends on the consistency of the set \( P \) of accepted sentences and the set \( M \) of atomic

\(^7\)As indicated in the previous footnote, I am considering here the notion of truth in a structure. The structure we are talking about is, of course, one of the \( A \)-normal extensions of \( A \).
sentences and negations thereof obtained from the relevant partial relations. However, in order to make \((\alpha \land \neg \alpha)\) quasi-true, we need an \(A\)-normal structure in which \((\alpha \land \neg \alpha)\) holds. But, of course, in a Tarskian type characterisation of truth, such as the one presented here, a contradiction cannot be true in a model.

Nonetheless, this is not the only possible characterisation of quasi-truth. We can extend this notion in such a way that even a contradiction can be quasi-true. One way of doing this is by redefining the notion of a partial relation. Instead of requiring that the \(R_1\) and \(R_2\) components of a partial relation \(R\) are mutually disjoint sets (such that \(R_1 \cap R_2 = \emptyset\)), we may say that some overlapping is accepted (making then \(R_1 \cap R_2 \neq \emptyset\)). In that case, a partial relation allows that some \(n\)-tuples satisfy both the \(R_1\) and \(R_2\) components. In this regard, since partial relations are usually thought of as representing ‘empirical’ information, ‘basic inconsistencies’ at the ‘empirical’ level can be accommodated. Of course, in order for this possibility to get off the ground, we need an appropriate paracompleteness logic as the underlying logic of the ongoing construction. Having one of these logics at hand — such as da Costa’s C-logics (see da Costa [1974] \(^8\) —, we can proceed to extend the notion of quasi-truth in a paracomplete setting.

Before outlining this construction, let us ask why should such an account be developed. One of the reasons is the following. As we saw, quasi-truth is weaker than truth, and to this extent it is more appropriate than the latter for accommodating several aspects of our actual epistemic situation. Moreover, since we face inconsistencies in so many contexts (from belief systems to scientific theories), and given that these inconsistencies cannot be dismissed without loss of information, nor can they be taken as true simpliciter, since this hardly corresponds to the judgments of the scientific community, the natural move is to countenance a notion of truth in which inconsistencies can be accommodated (for further discussions and arguments, see da Costa and French [1993] and [1998], Chapter 5). In a certain sense, the extant formulation of quasi-truth can take some sorts of inconsistencies into account — after all, the logic associated with this truth notion is paracomplete (see da Costa, Bueno and French [1998]). However, as we have just seen, contradictions of the form \((\alpha \land \neg \alpha)\) cannot be quasi-true. So, if we are to present an account in which both a weaker notion of truth can be countenanced and contradictions can be accommodated in an explicit way — such that \((\alpha \land \neg \alpha)\) can be quasi-true — a different characterisation of this latter notion needs to be formulated.

Here is a suggestion of how this can be done. The notions of partial structure and partial relation should first be revised in the way just mentioned in order to allow the introduction of ‘inconsistencies’ at the ‘basic level’. An ‘inconsistent partial relation’ \(R\) on a domain \(D\) is a triple \((R_1, R_2, R_3)\), where (1) \(R_1 \cup R_2 \cup R_3 = D^n\), (2) \(R_1\), \(R_2\), and \(R_3\) are as in the standard definition of partial relation, and (3) some overlapping is allowed between the \(R_1\) and \(R_2\) components, so that \(R_1 \cap

\(^8\) For a general formal framework that accommodates both quasi-truth and C-systems in terms of an account of belief change, see da Costa and Bueno [1997a] (see also da Costa and Bueno [1997b]).
The situation described in (3) may happen in borderline circumstances in which some 'fuzziness' is involved. Cases comprising the change of physical states come immediately to mind. Consider a quite simplistic example: a piece of ice melting on a table. Some people may claim, on the one hand, that there is no ice there any longer, since (at least part of) it became water. But on the other hand, given that there are still pieces of water in solid state on the table, others may claim that what we have there is ice. So there is and there is not ice on the table. A similar case is the one in which rain is stopping, so that according to some writers (such as Priest [1987]), it is raining and not raining.

In my view, borderline cases such as these, even if inconsistent, are not necessarily actual instances of 'true contradictions'. There are two reasons for this. Firstly, it is not at all clear that such cases can be adequately described by using the notion of truth; the information under consideration is just too fuzzy for that. Due to the fuzziness of the information involved, we cannot assume the existence of a perfect 'correspondence' between our description of the situation and the actual states of affairs, and so the talk of truth is beside the point. Secondly, in certain contexts, such cases can be accommodated more straightforwardly in terms of an appropriate fuzzy logic. Thus, instead of concluding that it is both raining and not raining, depending on the information we have about the domain under consideration, we may either conclude that it is raining, and reject the other alternative, or the other way around — but we do not conclude that both outcomes hold. This requires, of course, an assignment of weights to the information we have, so that a choice can be made between the two sides of the 'dilemma'. The 'fuzziness' is then dissolved by checking and choosing the relevant information.

However, it may be claimed that, in having to choose and evaluate the information, some arbitrariness may be introduced. ('Why assume a particular assignment of weights to the information?', 'Why reject this bit of information instead of the other?' are obvious questions.) Furthermore, there may not be conclusive grounds in terms of which a decision can be warranted. Thus, in order not to lose information about the domain under consideration, and not to rest

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9It may be allowed, of course, that \( R_1 \cap R_2 \neq \emptyset \) or \( R_2 \cap R_3 \neq \emptyset \), although this would bring a different sort of 'inconsistency', since it involves the possibility that for some \( n \)-tuples it is both not defined whether they belong to \( R \) and it is defined and they do not belong to \( R \) (in the case \( R_2 \cap R_3 \neq \emptyset \) — or it is so defined and they belong to \( R \) (in the case \( R_1 \cap R_3 \neq \emptyset \)). Thus, in these cases, we have, as it were, a double level of 'inconsistency'. Firstly, there is a 'definitional' inconsistency — between what is defined and what is not in a partial relation. Secondly, there is the 'inconsistency' between being and not being an element of \( R \) — which assumes, of course, that the \( n \)-tuples under consideration are defined either as elements of \( R \) or not. In what follows, I shall only consider the second level of 'inconsistency'.

10Notice that even if we had perfect information about the situation (for example, if we knew the number of rain drops that fell, exactly where they fell etc.), we might still be unable to avoid the inconsistency that it is raining and not raining. But what this indicates is that 'rain' is a vague predicate. And the issue then is whether we can combine the vagueness of the predicate with the notion of truth. If we are to claim that, in such borderline cases, the inconsistency is true, we need to be told how this combination is to be achieved. It is not enough to provide a Tarskian construction of truth (\textit{simpliciter}) in a paraconsistent setting (see Priest [1987], Chapter 9). An account of vagueness, adequate to this setting, is also required.
any choice on arbitrary grounds, a paraconsistent approach may be advisable. Nonetheless, given the provisional nature of the information under consideration, the assertions describing the situation are not taken to be true, and in this respect the talk of ‘true contradictions’ does not seem to be appropriate. What is needed, instead, is an open-ended framework which extends the notion of truth to inconsistent situations.

In my view, the best strategy to accommodate this demand is to provide a convenient reformulation of the notion of quasi-truth. After all, as we saw in the previous section, quasi-truth and partial structures supply a broad conceptual setting to address, in a more open-ended way, issues involving truth. The first step to be taken is to introduce the notion of an ‘inconsistent’ partial structure. The idea is to extend the concept of partial structure, as defined in the previous section, by allowing the introduction of ‘overlapping’ or ‘inconsistent’ partial relations (in the sense just spelled out). An ‘inconsistent’ partial structure is then a structure, in the usual sense, involving a domain $D$ of objects plus a family of relations defined on $D$, which includes some ‘overlapping’ partial relations.

The second step is then to reformulate the notion of normal structure. Given an ‘inconsistent’ partial structure $A$, we say that a structure $B$ is an $A$-quasi-normal structure if $B$ is an $A$-normal structure — and thus its relations are defined for every $n$-tuples of objects of its domain — and some of these relations ‘overlap’. (It is assumed that the ‘overlapping’ components in $B$ — that is, the relations $R_1$ and $R_2$ such that $R_1 \cap R_2 \neq \emptyset$ — contain all the ‘overlapping’ components in $A$, and possibly some others that arise from extending $R_3$ relations which are in $A$.)\(^{11}\) Of course, the existence conditions for quasi-normal structures are not the same as the ones for normal structures. As opposed to the latter, the former does not depend on the consistency between the set $P$ of accepted sentences and the set $M$ of atomic formulas and negations thereof. The crucial requirement here is the non-triviality of these constructions, in the sense that although $P$ and $M$ are inconsistent, we cannot derive all the sentences of the language from their conjunction. Therefore, the usual paraconsistent move from the avoidance of inconsistency to the avoidance of non-triviality is made at this level.

Thus far, we have been extending the main components of the partial structures framework in order to explicitly accommodate inconsistencies, but preserving as much as possible of its original formulation. We shall stick, as far as possible, to the same policy in defining quasi-truth. As we saw in the previous section, this notion was characterised in terms of Tarski’s account of truth. But this account, as is well known, is in general too restrictive with regard to inconsistencies. We will have then to avoid its use in extending the concept of quasi-truth, and adopt instead da Costa’s theory of valuation.\(^{12}\) This theory, whose details I shall not consider here, is a broad conceptual framework, in terms of which a general, abstract study of several logics can be provided. The

\(^{11}\) The truth conditions for sentences in $B$ are given below.

\(^{12}\) I owe this point to Newton da Costa. For further information about da Costa’s theory of valuation, see for instance da Costa and Béziau [1994], da Costa and Béziau [1996], Béziau [1997], and Grana [1990].
crucial notion is of a \textit{bivailuation}. It is a function from the set of propositions of the logic under consideration onto the set \{0, 1\}, and it is defined in such a way to generalise the standard notion of valuation. In terms of this framework, an explicitly inconsistent notion of quasi-truth can then be presented. Roughly speaking, a sentence \(a\) is 'inconsistently' quasi-true in an 'inconsistent' partial structure \(A\) if there is an \(A\)-quasi-normal structure \(B\) in which \(a\) is true (in the sense of da Costa's theory of valuation).\(^ {13}\)

In terms of this extended notion of quasi-truth, we can then explicitly accommodate inconsistencies in a broad, open-ended framework — as required.

5. Quasi-truth, dialetheism and the T-scheme

Having sketched this formal framework, let me spell out some of its features. First, does Tarski’s T-scheme hold for quasi-truth? In other words, do we have that

\[(3) \quad Q_T(\alpha) \leftrightarrow \alpha?\]

Quite independently of the nature of the biconditional used in (3) — whether it is a relevant or a material biconditional \(^ {14}\) —, the equivalence in question does not hold. On the one hand, if \(\alpha\) is quasi-true, it does not follow that \(\alpha\) is true, since it may not be true in some \(A\)-normal structure. So, it is possible that we have \(Q_T(\alpha)\) without having \(\alpha\). On the other hand, if \(\alpha\) is the case, then \(\alpha\) certainly is quasi-true, given that for some \(A\)-normal structure it is true. Quasi-truth requires, thus, a reformulated version of Tarski’s scheme, namely:

\[(4) \quad \alpha \rightarrow Q_T(\alpha)\]

Of course, for several writers, the T-scheme is crucial for a notion of truth — or even, more strongly, for truth itself. Without this scheme, or so it is argued, we cannot claim to be dealing with truth, nor even with a truth notion (for a discussion, see Priest [1987], pp. 21-23; 69-74). Let us grant, for the sake of argument, that the T-scheme is a crucial property of a notion of truth. It is a feature of a strong — perhaps too strong — truth notion. However, there is no reason why a weaker characterisation of truth should satisfy the T-scheme. This is, in fact, one of the basic aspects of quasi-truth. The idea is that, despite being a truth notion — and it actually is an extension of Tarski’s account, coinciding with the latter when we consider full structures —, the non-satisfaction of the T-scheme represents an important trait of this notion: the accompanying counterpart to its weakness. As was seen, quasi-truth is not sufficient for truth. This point brings then the issue: what would be sufficient?

Several answers could be presented here, but I am going to consider only one. If a sentence \(a\) is true in all \(A\)-normal structures, then it is true. It

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\(^{13}\)The details of this formulation and its connection with da Costa’s C-systems will be developed elsewhere (see da Costa and Bueno [1997b]). On how to close truth-value gluts using supervaluation ideas, see Varzi [1995].

\(^{14}\)For a discussion of this issue, see Priest [1987], pp. 74-77; 89; 104-107.
is clear then that a stronger condition can be added to quasi-truth in order for it to lead to a characterization of truth. Let us call this the universality condition, and let us denote by \(Q_{TU}(\alpha)\) the statement that \(\alpha\) is quasi-true in all \(A\)-normal structures. Now, let us suppose that \(B_\emptyset\) is the \(A\)-normal structure which represents complete information about the actual world. Thus, we can also formulate the condition that a sentence is quasi-true with respect to \(B_\emptyset\). Let us call this the actuality condition, and let us denote by \(Q_{TA}(\alpha)\) the statement that \(\alpha\) is quasi-true with respect to \(B_\emptyset\) (in other words, \(\alpha\) is true in the \(A\)-normal structure which represents complete information about the actual world). We have, therefore, the following results:

\[
T(\alpha) \iff \alpha
\]

(5)

\[
\alpha \rightarrow Q_T(\alpha)
\]

(6)

\[
Q_{TU}(\alpha) \rightarrow \alpha^{15}
\]

(7)

\[
\alpha \iff Q_{TA}(\alpha).
\]

(8)

These equivalences and implications lead then to interesting results with regard to the relationship between these operators. By transitivity, from (5) and (8) it follows that

\[
T(\alpha) \iff Q_{TA}(\alpha).
\]

(9)

And so, we regain, as expected, Tarski’s scheme for a particular kind of quasi-truths, namely those sentences which are true in the \(A\)-normal structure \(B_\emptyset\) (satisfying the actuality condition). For those sentences which do not meet this condition, all we can claim is, at best, the implication (6), or if the universality condition is satisfied, (7).

Two considerations should be explored at this point. Since we are concerned with accommodating a notion of truth in inconsistent contexts, in what sense can we claim that truth is inconsistent? Moreover, if there is such a sense, can it be distinguished, in any interesting way, from the existence of true contradictions? These considerations will be taken in turn.

What is the relationship between the inconsistency of truth and the existence of truth contradictions? The claim that truth is inconsistent can be presented as follows:

\[
T(\alpha) \land \neg T(\alpha).
\]

(10)

\[^{15}\text{Notice that the converse of this statement is not true, since the fact that } \alpha \text{ is the case does not entail that it is true in all } A\text{-normal extensions. I am grateful to Priest for pointing this out to me.} \]
committed to the distinction between truth being inconsistent and the existence of true contradictions. (Since this depends on the non-contraposibility of the conditional in question, if we accept the contraposibility, the equivalence will hold. Another route to obtain the equivalence is to change the truth operator to another, perhaps one of quasi-truth.) In any case, since (10) and (11) are independent, the acceptance of one does not imply the acceptance of the other.

This distinction, however, cannot be made without some consequences for dialetheism. Here are some of them. (A) Because of the distinction between (10) and (11), it is possible for someone to claim that truth may be inconsistent without having, ipso facto, to countenance the existence of true contradictions. Moreover, the acceptance of (10) does not make someone a dialetheist. This consequence is far from being desirable, given that the point of believing in true contradictions, I take it, is to reach new truths; but we cannot conclude anything about the nature of these truths given the distinction between (10) and (11). In this respect, the concern of dialetheism with truth and inconsistency is not appropriately taken into account. (B) Furthermore, dialetheism becomes a weak proposal. It is already metaphysically committed to a particular notion of truth (namely, Tarski's plus a suitable paraconsistent construction), according to which contradictions can be true, but this very notion, at least as currently formulated, is not itself necessarily 'inconsistent' (or, at least, dialetheism does not entail this). The whole position is weakened with the distinction, since dialetheism does not entail the stronger claim about the inconsistent nature of truth — or, at least, concerning a feature of the truth operator that seems crucial for those who argue for the existence of true contradictions.

(C) Moreover, faced with this distinction, it seems arbitrary to characterise dialetheism as the doctrine according to which there are true contradictions (see Priest [1987], p. 4), if one is actually concerned with the inconsistent features of truth. After all, I take it that the project is not simply to provide a framework in which inconsistencies (even true ones) can be accommodated (this is entirely compatible with an agnostic view about inconsistent truths). But the point is to argue that the notion of truth, as a crucial component of inquiry, plays a crucial role in such a framework. And in order to do so, we need more than 'dialetheias' (true contradictions), we need an appropriate inconsistent notion of truth (such as the one supplied by (10)). Indeed, in the extant formulation of dialetheism, the role played by truth in characterising true contradictions

\[ T(\alpha) \land \neg T(\alpha) \]

is to accept a contradiction as true. Moreover since \( \beta \rightarrow T(\beta) \), (16) entails something of the form \( T(\gamma \land \neg \gamma) \). I think we have to be careful here. To be explicit, what is stated in (16) is that for some sentence \( \alpha \), \( T(\alpha) \land \neg T(\alpha) \). The logical form of (16) is then \( \exists \alpha (T(\alpha) \land \neg T(\alpha)) \). Therefore, what (16) entails is that

\[ T(\exists \alpha (T(\alpha) \land \neg T(\alpha))) \]

and so we don't have something of the form \( T(\gamma \land \neg \gamma) \). As a result, it is not enough to accept that truth is inconsistent to become a dialetheist.
The idea is, of course, that the truth operator has inconsistent features. The existence of true contradictions, the basic claim of Priest’s dialetheist proposal (see his [1987]), can be formulated in the following way:

\[(11) \quad T(\alpha \land \neg \alpha).\]

From the T-scheme we have that:

\[(12) \quad T(\neg \alpha) \leftrightarrow \neg \alpha,\]

and assuming that the conditional in the T-scheme is contraposible, we can derive

\[(13) \quad \neg T(\alpha) \leftrightarrow \neg \alpha.\]

It then follows from (12) and (13), by transitivity, that

\[(14) \quad \neg T(\alpha) \leftrightarrow T(\neg \alpha).\]

Now, if we suppose that the T-scheme satisfies the following equivalence (the ‘linearity’ of the conjunction with respect to the truth predicate):

\[(15) \quad T(\alpha \land \beta) \leftrightarrow T(\alpha) \land T(\beta),\]

from (10) we derive (11), and vice-versa (using, of course, the equivalence (14)). In other words, assuming the contraposibility of the conditional in the T-scheme, and the ‘linearity’ of the conjunction, there is no (extensional) distinction between the truth being inconsistent and the existence of truth contradictions.

This seems to be at least philosophically appropriate in the following sense. Those who countenance the existence of true contradictions should accept that at least some truths are inconsistent — and in this respect, truth is inconsistent. However, according to Priest, this is not the case. Relying on the principle that contradictions should not be multiplied beyond necessity (Priest [1987], pp. 90 and 144-145), he claims that the move that leads from (11) to (10) — leading from an ‘internal’ to an ‘external’ inconsistency — should be rejected. Despite countenancing, as a dialetheist, inconsistencies at all levels — both at the so called ‘object-theory’ and at the ‘metatheory’ (ibid., pp. 88-89)\(^\text{16}\) —, he thinks that principles that lead to the multiplication of inconsistencies should be restricted. As a result, Priest insists that the conditional in the T-scheme is not contraposible (ibid., pp. 99-100). Thus, (13) and (14) do not hold, and the ‘spread’ of inconsistency is avoided. Hence, in Priest’s account, (10) and (11) turn out not to be equivalent (ibid., pp. 88-91), and therefore his account is

\(^{16}\)In Priest’s view, the distinction between object-theory and metatheory is spurious, motivated by ‘incorrect attempts to impose consistency’ (Priest [1987], p. 89). By rejecting this distinction, he argues that the semantics of a natural language, or of a formal language for that matter, can be given within the very language in question (for a construction to this effect, see ibid., pp. 157-166; 172-177). It goes without saying that, provided that an appropriate paraconsistent logic is adopted in this context, the resulting inconsistencies shall not lead to triviality (for a discussion, see ibid., pp. 165-171).
is, in many respects, subsidiary. The ‘crucial work’, as it were, comes from the inconsistency side. After all, we have first to produce a ‘contradiction’ and then claim that it is true. In this sense, the fact that dialetheism does not imply (i) points to a certain ‘incoherence’ in the programme. It delivers true contradictions, where one expects to have inconsistent truths. (I take it that we are searching for the truth, and not for contradictions!) And given the distinction between (10) and (11), these are not quite the same.\footnote{It might be claimed that the Liar sentence, ‘this sentence is not true’, gives a contradiction of the form (10). I agree with this. The issue, however, is whether the dialetheist, only with the acceptance of (11), is entitled to assert (10). As we saw, with the rejection of a contraposible T-schema, this is not the case. As a result, in order to obtain (10), the dialetheist has to introduce a further, ‘empirical’ premise (such as the Liar) to obtain an inconsistent notion of truth. To say the least, as an account of truth, this is clearly ad hoc.}

At this point, a stronger paraconsistent approach, which countenances (10), instead of (11), as the basic feature of the proposal, can be articulated. The idea is to countenance the existence of inconsistent truths, or rather that truth may be inconsistent, so that in (10) there is an elliptical existential quantification over a, namely $T(a) \land \neg T(a)$, for \textit{some} a. Of course, by assuming a contraposible conditional in the T-schema, this account would imply the dialetheist principle (11), and in this respect, it is stronger than dialetheism.

Notice that the dialetheist view does not assume a contraposible conditional in the T-schema. Otherwise, it would be ‘trivialised’ into a classical account of truth, delivering both the exhaustion principle

\begin{equation}
- T(a) \rightarrow T(\neg a),
\end{equation}

to which the dialetheist is committed, and the exclusion principle

\begin{equation}
T(\neg a) \rightarrow \neg T(a),
\end{equation}

that can only be avoided by countenancing non-contraposible conditionals (see Priest [1987], pp. 88-91; 99-100).\footnote{In correspondence, Priest indicated that ‘a dialetheist can accept a contraposible T-schema’. In his view, ‘this does not deliver the classical account of truth at all — if this means a consistent one: truth may still be inconsistent. The negation may still be a paraconsistent one’. I agree that the dialetheist can adopt a Tarskian definition of truth (\textit{simpliciter}), and change the underlying logic to a paraconsistent one. That’s essentially what Priest has done in Chapter 9 of In Contradiction (Priest [1987]). My difficulty here is to reconcile Priest’s claim that ‘a dialetheist can accept a contraposible T-schema’ with the arguments he presented in In Contradiction ‘against the contraposibility of the T-schema’ (Priest [1987], p. 100). According to Priest ([1987], p. 99):

The validity of the T-schema is essentially the condition:

\begin{equation}
1 \in v(a) \iff a \in d^+(T)
\end{equation}

where $a$ is any closed sentence $v$ is an evaluation function, which maps formulas of the language to the set of truth values $\pi = \{ \{0\}, \{1\}, \{0, 1\} \}$, and $d^+(T)$ is the extension of the truth predicate T; see Priest [1987], pp. 94-98.}

For closed $\alpha$ the exhaustion principle, $\neg T_\alpha \rightarrow T_{\neg \alpha}$, should also be validated. This is essentially the condition:
approach accepts a contraposible conditional in the T-scheme, why is it not similarly committed to this classical account? The answer, in a nutshell, is that this paraconsistent account will also change the notion of truth. The idea is, of course, to take the reformulation of quasi-truth sketched in the previous section as the underlying truth notion. And since this extended notion of quasi-truth can explicitly accommodate inconsistencies, and it is sufficiently open-ended to allow an adequate account of several aspects of inquiry, it seems to be an appropriate notion to take. Moreover, as the considerations presented in the beginning of this section point out, the T-scheme holds for quasi-truth, provided that the actuality condition obtains.\(^\text{20}\)

As an illustration, let us consider what can be called the quasi-Liar sentence: `this sentence is not quasi-true’. Now, if the quasi-Liar sentence is indeed not quasi-true, then what it says (that it is not quasi-true) is true, and so there is a structure in which it is true. Therefore, the sentence is quasi-true. On the other hand, let us suppose that the quasi-Liar sentence is quasi-true. Since what it says is that it is not quasi-true, there is no structure in which it is true. Therefore, the sentence is not quasi-true. Thus, the quasi-Liar is quasi-true iff it is not quasi-true, i.e. \(Q_T(\alpha) \leftrightarrow \neg Q_T(\alpha)\). As a result, in the present account, it may well be the case that \(Q_T(\alpha) \land \neg Q_T(\alpha)\).

To sum up. The idea that truth is inconsistent can be clearly characterised in terms of (10). Dialetheism distinguishes this property of truth from the claim that there are true contradictions (which is spelled out in terms of (11)).\(^\text{21}\)

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\(\text{If } \alpha \in d^\sim (T) \text{ then } 0 \in v(\alpha)\) \quad (2)

(where \(d^\sim (T)\) is the anti-extension of the truth predicate \(T\). Should we require the converse condition?

\(\text{If } 0 \in v(\alpha) \text{ then } \alpha \in d^\sim (T)\) \quad (3)

This is essentially the exclusion principle [namely, \(T \neg \alpha \rightarrow \neg T \alpha\)], and the answer would seem to be ‘no’. \((\text{Priest [1987], p. 99})\)

After spelling out the reasons why we should not accept the exclusion principle (ibid., pp. 99-100), Priest concludes that ‘we should require only (1) and (2) of the truth predicate’ (ibid., p. 100). In other words, only the validity of the T-scheme and the exhaustion principle are required, but not the exclusion principle. After all, as Priest indicates, with the exclusion principle, ‘the truth predicate is [...] a partial consistencier’ (ibid.). So, the exclusion principle should not be accepted. However, Priest continues, ‘if the T-scheme were fully contraposible then (3) [the exclusion principle] would hold’ (ibid.). After showing how to derive the exclusion principle assuming a contraposible conditional (ibid.), Priest concludes: ‘these are the considerations I referred to [...] against the contraposibility of the T-scheme’ (ibid.; the italics are mine). For these reasons, I claim that the dialetheist does not accept a contraposible T-scheme.

But perhaps Priest has a modal distinction in mind. Although (a) the dialetheist does not accept a contraposible T-scheme in In Contradiction, (b) he can accept it (or could have accepted it). In this case, the dialetheist owes us at least an explanation of how to reconcile these two claims; in particular, if he can accept a contraposible T-scheme, why provide all these arguments against it?

\(^\text{20}\)It goes without saying that the underlying logic of the whole construction will have to be paraconsistent. We adopt da Costa’s C-logics (see da Costa [1974]).

\(^\text{21}\)It is not straightforward to understand the meaning of the claim that there are true
by doing so, it loses a stronger case for an inconsistent account of truth; an
account which can be provided in terms of the broader notion of (inconsistent)
quasi-truth suggested here.

The crucial point, however, is that quasi-truth, being strictly weaker than
truth, does not necessarily commit us to a complete description of the domain
of knowledge we are investigating. Therefore, it opens up the way for an agnostic
reading of the nature of the inconsistencies under consideration. In this respect,
we are back to the agnosticism from which we started. The idea is, of course,
to take the inconsistent models suggested here as nothing but tools; tools to
investigate the domain of inconsistent, and at best partially described, truths.²²

6. References

1. Béziau, J.-Y., Théorie de la valuation, appendix to Newton da Costa, Logiques
Sc. 28 (1997), 585-610.
3. _____, What is Structural Empiricism? Scientific Change in an Empiricist Set-

contradictions; in particular, when such contradictions are not taken as features of a conceptual
system, such as mathematics, but are 'about the world'. If we are only considering conceptual
systems, we may well produce 'true contradictions', provided that we have an appropriate
paraconsistent set theory. In this setting, we can prove, for instance, that Russell's set is and
is not a member of itself (see da Costa [1986b], and da Costa, Béziau and Bueno [1996]).
But how should we understand true contradictions about the world? According to Priest
[1987], such contradictions are those whose predicates refer to physical objects, and they arise
especially in cases involving changes in these objects. Notice that this is weaker than the
claim to the effect that there are contradictions in reality (whatever this means), since the
contradictions discussed by Priest are features of our language, and not necessarily of the
world. What is inconsistent are our theories about the world, not the world itself; what is
true are our claims about the world, and not the world. So, even if true statements require
language and the world, why should the fact that we have advanced an inconsistent theory
about the world imply that the world itself is 'inconsistent'? Moreover, to the best of my
knowledge, I don't know of any true inconsistent theory about the world. In science, an
inconsistent theory (such as, Bohr's theory) is not taken to be true, but at best quasi-true
(see da Costa and French [1993a]).

Suppose, however, that we want to develop a stronger (ontic, rather than semantic) version
of these remarks. In order to do so, as Priest noticed (in a personal communication), we may
have to talk about inconsistent facts. In this sense, we would need perhaps a sort of 'neo-
Tractarian' approach; an approach according to which facts are not logically independent, but
could conflict one with the others. (In this respect, the picture would certainly differ from
the one presented by Wittgenstein in the Tractatus.) Consider, for instance, the possibility
that certain objects could be both green and red, and that this could be a fact. In this case,
the kind of inconsistency in the world could have been presented. The problem, of course, is
how to develop this alternative in a sensible way. Perhaps the sort of modelling provided in
the previous section by the inconsistent account of partial structures could be used in this
context. However, I won't dare to develop this suggestion here.

²² Many thanks to Newton da Costa, Steven French and Graham Priest for helpful and
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improvements. Thanks are also due to an anonymous referee for similarly perceptive remarks,
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