Quasi-Truth, Supervaluations and Free Logic

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The partial structures approach has two major components: a broad notion of structure (partial structure) and a weak notion of truth (quasi-truth). In this paper, we discuss the relationship between this approach and free logic. We also compare the model-theoretic analysis supplied by partial structures with the method of supervaluations, which was initially introduced as a technique to provide a semantic analysis of free logic. We then combine the three formal frameworks (partial structures, free logic and supervaluations), and apply the resulting approach to accommodate semantic paradoxes.

1. Introduction

In 1986, Mikenberg et al. put forward a generalization of the Tarskian account of truth in terms of partial structures and quasi-truth (see Mikenberg et al. 1986; da Costa, 1986).1 The main idea was to provide a framework in which it was possible to accommodate the partiality of information found in scientific domains. For example, given that we often lack complete information about a particular domain of inquiry, we may be unwarranted in claiming that a particular theory about this domain is true. But we may well claim that, as far as our current information is concerned, the theory may be true; that is, nothing thus far precludes that the theory is indeed true (in the domain under consideration).

In order to articulate this idea, two crucial notions were introduced:

1. A broad concept of structure (partial structure), which represents the partial information about the domain in question.
2. A weak notion of truth (quasi-truth), which accommodates the accompanying epistemic judgement.

In terms of these notions, a distinctive approach has been applied to a number of issues in the philosophy of science, inductive logic and the philosophy of mathematics.2

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1 Quasi-truth was initially introduced as pragmatic truth, due to the connection between the formalism provided by partial structures and some claims made by pragmatist philosophers about the notion of truth (see Mikenberg et al., 1986, see also da Costa and French, 1999, Chapter 1). But it was soon realized that pragmatic truth was only a particular interpretation of the formalism; the latter could also be interpreted as an epistemic possibility of truth. If a sentence T is pragmatically true, the current information doesn't prevent that T is indeed true (see da Costa et al., 1998, and Bueno and de Souza, 1996). To make room for the different interpretations, the neutral notion of quasi-truth has been introduced.

In 1988, Corcoran wrote a review of Mikenberg et al. (1986) for Mathematical Reviews (Corcoran 1988). After presenting the content of the paper in an insightful way, Corcoran noticed:

At certain points in the philosophical discussion the paper implicitly distinguishes between partial knowledge about a total structure and total knowledge about a partial structure. It also implies that pragmatic thinking construes an incomplete set of beliefs as being about a partial structure. This suggests that pragmatic thinking takes place in a partially interpreted language and thus presupposes a free logic.

Corcoran couldn’t be more right. There is indeed a close connection between the partial structures approach and free logic (and, in particular, with the method of supervaluations as a technique to supply a semantics for free logic). The aim of the present paper is to explore this connection.

After indicating the main features of the partial structures approach in section 2, we shall briefly present, in section 3, the main points of free logic and supervaluations. In section 4, a combined framework is then presented, which puts together partial structures, free logic and supervaluations. Finally, in section 5, an application of the resulting framework to the semantic paradoxes is discussed.

2. The partial structures approach

The partial structures approach relies on three main notions: partial relation, partial structure and quasi-truth. This proposal is introduced to provide a formal framework in which the ‘openness’ and ‘incompleteness’ of information dealt with in scientific contexts can be uniformly accommodated (see da Costa and French, 1990, 1999). This is accomplished by first extending the usual notion of structure; in order to model the partialness of information about a domain, the notion of a partial structure is introduced. Secondly, the Tarskian characterization of truth is generalized for ‘partial’ contexts – and the corresponding concept of quasi-truth is formulated.

But the notion of partial structure can only be introduced after we formulate an appropriate notion of partial relation. When a domain of knowledge $\Delta$ is investigated, we put forward a conceptual framework to systematize and organize the information about $\Delta$. This domain is tentatively represented by a set $D$ of objects, and is studied by the examination of the relations holding among $D$’s elements. The problem is that it often happens that, given a certain relation $R$ defined over $D$, we do not know whether all the objects of $D$ (or $n$-tuples thereof) are related by $R$. This is part and parcel of the ‘incompleteness’ of our information about $\Delta$, and is formally accommodated by the concept of partial relation. More formally, let $D$ be a non-empty set. An $n$-place partial relation $R$ over $D$ is a triple $\langle R_1, R_2, R_3 \rangle$, where $R_1$, $R_2$ and $R_3$ are mutually disjoint sets, with $R_1 \cup R_2 \cup R_3 = D^n$, and such that: $R_1$ is the set of $n$-tuples that (we know that) belong to $R$, $R_2$ is the set of $n$-tuples that (we know that) do not belong to $R$, and $R_3$ is the set of $n$-tuples for which we do not know whether they belong or not to $R$. (Notice that if $R_3$ is empty, $R$ is a usual $n$-place relation which can be identified with $R_1$.)

However, in order to represent the information about the domain under study, we need a notion of structure. The following characterization, spelled out in terms of partial relations and based on the standard concept of structure, is meant to provide a notion which is broad enough to accommodate the partiality usually found in scientific practice. The main work is done, of course, by the partial relations. A partial
structure $A$ is an ordered pair $\langle D, (R_i)_{i \in I} \rangle_{\alpha \beta}$, where $D$ is a non-empty set, and $(R_i)_{i \in I}$ is a family of partial relations defined over $D$.3

In the original formulation of the partial structures approach, the notion of a partial structure was formulated in a more fine-grained way (see da Costa, 1986; Mikenberg et al., 1986). In order to systematize our knowledge of $\Delta$ (say, the physics of particles), the domain $D$ of the partial structure $A$ is typically constituted by two components:

(1) Observable, 'actual' objects (in the physics of particles, configurations in a Wilson chamber, spectral lines etc.), whose set is denoted by $D_1$.
(2) Unobservable, 'non-actual' objects (quarks, for example), whose set is denoted by $D_2$.

It is understood that $D_1 \cap D_2 = \emptyset$, and it is required that $D = D_1 \cup D_2$. In this way, the modelling of $\Delta$ involves new partial relations $R_i, i \in I$ (defined over $D_2$), some of which may help to extend the relations $R_i, i \in I$ (defined over $D_1$). As a result, if we want to be explicit, a partial structure $A$ has the following form: $\langle D_1, D_2, R_i, (R_i)_{i \in I} \rangle_{\alpha \beta}$, But it is usually easier to refer to it simply as $\langle D, (R_i)_{i \in I} \rangle_{\alpha \beta}$ (see also da Costa and French, 1990, and da Costa et al., 1998).

Two of the three basic notions of the partial structures approach are now defined. In order to spell out the last, and crucial one – quasi-truth – an auxiliary notion is required. The idea is to use, in the characterization of quasi-truth, the resources supplied by Tarski’s definitions of truth. However, since the latter is only defined for full structures, we have to introduce an intermediary notion of structure to ‘link’ full to partial structures. The structures which ‘extend’ a partial structure $A$ into a full, total structure are called $A$-normal structures. Their major role is purely model-theoretic; namely, to put forward an interpretation of the language used to make claims about $\Delta$ and, in terms of this interpretation, to characterize basic semantic notions. But how should we characterize $A$-normal structures? Here is an answer. Let $A = \langle D, (R_i)_{i \in I} \rangle_{\alpha \beta}$ be a partial structure. We say that the structure $B = \langle D', (R'_i)_{i \in I} \rangle_{\alpha \beta}$ is an $A$-normal structure if:

(i) $D = D'$, (ii) every constant of the language in question is interpreted by the same object both in $A$ and in $B$, and (iii) $R'_i$ extends the corresponding relation $R_i$ (in the sense that each $R'_i$, supposed of arity $n$, is defined for all $n$-tuples of elements of $D'$). Notice that, although each $R'_i$ is defined for all $n$-tuples over $D'$, it holds for some of them (the $R'_{i\alpha}$-component of $R'_i$), and it doesn’t hold for others (the $R'_{i\beta}$-component).

As a result, given a partial structure $A$, there are several $A$-normal structures, and we need to provide constraints to restrict the acceptable extensions of $A$. In order to do that, we need a further auxiliary notion (see Mikenberg et al., 1986). A pragmatic structure is a partial structure to which a third component has been added: a set of accepted sentences $P$, which represents the accepted information about the structure’s domain. (Depending on the interpretation of science which is adopted, different kinds of sentences are introduced in $P$: realists will typically include laws and theories, whereas empiricists will add certain laws and observational statements about the domain in question.) A pragmatic structure is then a triple $A = \langle D, (R_i, P)_{i \in I} \rangle_{\alpha \beta}$, where $D$ is a non-empty set, $(R_i)_{i \in I}$ is a family of partial relations defined over $D$, and $P$ is a set

3 Notice that the partiality described here is due to the ‘incompleteness’ of our knowledge about the domain under investigation. Given further information about this domain, a partial relation may become total. Hence, we are concerned here with an epistemic, not an ontological partiality.
of accepted sentences. The idea is that $P$ introduces constraints on the ways that a partial structure can be extended (the sentences of $P$ hold in the $A$-normal extensions of the partial structure $A$).

Given a pragmatic structure, one condition for the existence of $A$-normal structures can now be spelled out (see Mikenberg et al., 1986). Let $A = \langle D, R, P \rangle_{et}$ be a pragmatic structure. For each partial relation $R_n$, we construct a set $M_n$ of atomic sentences and negations of atomic sentences, such that the former correspond to the $n$-tuples which satisfy $R_n$, and the latter to those $n$-tuples which do not satisfy $R_n$. Let $M = \bigcup_{n} M_n$. Therefore, a pragmatic structure $A$ admits an $A$-normal structure if, and only if, the set $M \cup P$ is consistent.

Assuming that this condition is met, we can now formulate the concept of quasi-truth. A sentence $\alpha$ is quasi-true in a pragmatic structure $A = \langle D, R, P \rangle_{et}$ if there is an $A$-normal structure $B = \langle D', R', P \rangle_{et}$ such that $\alpha$ is true in $B$ (in the Tarskian sense). If $\alpha$ is not quasi-true in $A$, we say that $\alpha$ is quasi-false in $A$. Moreover, we say that a sentence $\alpha$ is quasi-true if there is a pragmatic structure $A$ and a corresponding $A$-normal structure $B$ such that $\alpha$ is true in $B$ (according to Tarski's account). Otherwise, $\alpha$ is quasi-false.

The idea, intuitively speaking, is that a quasi-true sentence $\alpha$ does not describe the whole domain it is about, but only an aspect of this domain – the one modelled by the relevant partial structure $A$. After all, there are several different ways in which $A$ can be extended to a full structure, and in some of these extensions $\alpha$ may not be true. As a result, the notion of quasi-truth is strictly weaker than truth: although every true sentence is (trivially) quasi-true, a quasi-true sentence is not necessarily true (since it may be false in certain extensions of $A$).

Thus, as da Costa and French have pointed out (1990, 1993a), the notion of quasi-truth can be understood as an as if concept – if a sentence $\alpha$ is quasi-true, we can say that it describes the particular domain under consideration as if its description were true. Indeed, since $\alpha$ is consistent with the accepted information about the domain (represented by the set $P$), $\alpha$ is compatible with our knowledge about the domain. However, this doesn't commit us to believe that the extensions of the accepted information (formulated by the total $A$-normal structures) are true, since $\alpha$ may be false in some such extensions.4

It should be noticed that full structures can be ultimately dispensed with in the formulation of quasi-truth. For quasi-truth can also be characterised independently of the standard Tarskian type account of truth (see Bueno and de Souza, 1996). In this way, full, $A$-normal structures are entirely inessential; their use here is only a convenient device. Thus, the partial structures approach doesn't have to assume a primitive notion of modality (such as consistency) as part of its framework. (This remark will be important for our discussion below.)

To illustrate the use of quasi-truth, let us briefly consider an example. As is well-known, Newtonian mechanics is appropriate to explain the behaviour of bodies under certain conditions (roughly speaking, if the speeds in question are 'low' in comparison with that of light, the bodies are not subject to strong gravitational fields, etc.). And with the formulation of special relativity, we learnt that if these conditions are not satisfied, Newtonian mechanics is false. In this sense, these conditions specify a family of partial relations, which delimit the context in which the theory holds. Although Newtonian mechanics is not true (and we know under what conditions it is false), it is

4 Further details on logical and philosophical aspects of the notion of quasi-truth can be found in da Costa et al., 1998.
quasi-true; that is, it is true in a given context, determined by a pragmatic structure and a corresponding A-normal one, which satisfy the conditions mentioned above (see da Costa and French, 1993a).

Having presented the main components of the partial structures approach, we shall now consider the main points of free logic and the method of supervaluations.

3. Supervaluations and free logic

If we disregard certain anticipations in the first half of the twentieth century, the genesis of free logic can be traced back to a seminal paper by Leonard (see Leonard, 1956; for a discussion, see Lambert, 1991a and van Fraassen, 1991). As Leonard indicates, logic always has certain presuppositions, and one can try to remove them in order to extend logic’s domain of application. For example, as is well-known, in the traditional square of opposition, the general terms are taken to have existential import. So from the A sentence ‘All S is P’, one infers the I sentence ‘Some S is P’. This inference is rejected by contemporary logic; it is only accepted if it is assumed that there are some S’s. Of course, according to Leonard, this doesn’t mean that traditional logic is wrong. It only indicates a limitation of its domain of application (it can’t be applied to sentences involving non-denoting general terms). And once we are aware of this limitation, we can overcome it.

However, as Leonard argues, even contemporary logic has its own limitations. The existential import is no longer found in general terms, but it is transferred to singular ones. This becomes clear if we consider the inference schemes of Existential Generalization (EG) and Universal Instantiation (UI):

\[ Pa; \text{ therefore } \exists x \, Px \]
\[ \forall x \, Px; \text{ therefore } Pa \]

Now, if the term ‘a’ doesn’t denote (if it is, say, ‘Sherlock Holmes’), the schemes above seem to generate invalid inferences. This raises the question of how to construct a logic which doesn’t assume an existential presupposition even for its singular terms – and, accordingly, which indicates the terms for which EG and UI validly apply. But how should this be done?

Free logic was devised exactly to address this problem, and it is the result of the work of a number of people (see, for example, the selections in Lambert, 1991b, and references quoted therein). The term ‘free logic’ was coined by Lambert, and refers to a logic whose terms (both general and singular) are free of existential presuppositions, but whose quantifiers have existential force. In particular, inferences such as EG and UI are only valid if the term ‘a’ denotes. Various proof-theoretic systems of free logic have been provided, and we shall not review them here (see, for instance, Bencivenga, 1986, pp. 382–7, and Lambert, 1991a, pp. 6–9). The crucial idea is, of course, to make explicit the existential assumptions found in classical logic, and to dehistoric the cases in which such assumptions are justified from those in which they aren’t.

For our present purposes, it is enough to focus on the semantics of free logic. There are two major pictures associated with it (see Lambert, 1991a, p. 9; the following terminology was put forward by Scales). According to the Meinongian picture, the interpretation function introduced by the semantics is total; a singular term always has a value, which is either an existent or a non-existent object. As a result, all singular terms have reference. According to the Russellian picture, the interpretation function is partial; thus some singular terms lack reference.
More formally, a Meinongian picture can be formulated in the following way (see Lambert, 1991a, p. 10). Let \( S = \langle D_0, D_1, f, \rangle \) be a structure, where \( D_0 \) is a (possibly empty) set of objects, called the outer domain; \( D_1 \) is also a (possibly empty) set of objects, called the inner domain; \( D_0 \) and \( D_1 \) are disjoint, and their union is not empty; finally, \( f \) is the interpretation function such that: (a) if \( a \) is a term, \( f(a) \) is an element of the union of \( D_0 \) and \( D_1 \); (b) if \( P \) is an \( n \)-place predicate, \( f(P) \) is a set of \( n \)-tuples of members of the union of \( D_0 \) and \( D_1 \); and (c) every element of the union of \( D_0 \) and \( D_1 \) has a name. The intuitive idea is that \( D_0 \) is the set of non-existent objects, whereas \( D_1 \) is that of the existent ones. Now, a statement of the form \( P^a a_1, \ldots, a_n \) is True in \( S \) if \( \langle f(a_1), \ldots, f(a_n) \rangle \in f(P^a) \); otherwise it is False in \( S \). And the usual truth-conditions apply to the primitive connectives. Finally, a statement of the form \( \forall x B \) is True in \( S \) if \( B[a/x] \) is True in \( S \) for all \( a \) such that \( f(a) \in D_1 \); otherwise, it is False in \( S \). Of course, the crucial difference in this truth definition is the quantifier clause, where \( \forall x B \) is defined only on the singular terms which have values in the set of existent objects \( (D_1) \).

The Russellian picture is formally very similar to the Meinongian (Lambert, 1991a, p. 11). Let \( S' = \langle D, f \rangle \) be a structure, where \( D \) is a (possibly empty) set of objects (the existent ones), and \( f \) is an interpretation function such that: (a') where \( f(a) \) is defined, \( f(a) \) is an element of \( D \); (b') if \( P^a \) is an \( n \)-place predicate, \( f(P^a) \) is a set of \( n \)-tuples of members of \( D^a \); (c') every element of \( D \) has a name. A statement of the form \( P^a a_1, \ldots, a_n \) is True in \( S' \) if each of \( f(a_1), \ldots, f(a_n) \) are defined and \( \langle f(a_1), \ldots, f(a_n) \rangle \in f(P^a) \); otherwise, if any of \( f(a_1), \ldots, f(a_n) \) are not defined (don’t revert to an existent object), then the statement is false in \( S' \). The connective and quantifier clauses are the same as the Meinongian picture. (For further details and adequacy conditions for this semantics, see Lambert, 1991.)

Of course, both the Meinongian and the Russellian approaches face criticisms, and we should consider these shortly. For the moment, we only wish to outline such approaches, prior to exploring the connections with the partial structures formalism, but before moving to that, let us consider the second topic of the present section: supervaluations.

The method of supervaluation was initially devised to provide a semantic analysis of free logic (see van Fraassen, 1966a, 1966b, 1968, and 1969, see also Woodruff, 1984, 1991, and Bencivenga, 1986). The main idea is to accommodate (1) the introduction of truth-value gaps resulting from the language’s non-denoting terms, and (2) the requirement that logical truths preserve their truth-value even if they incorporate non-denoting terms. For example, if ‘\( b \)’ is a non-denoting term, it is intuitive to think that ‘\( Pb \)’ lacks truth-value; but one still wants to maintain that ‘\( Pb \lor \neg Pb \)’ is true (for a discussion, see Bencivenga, 1986, pp. 400–12). Van Fraassen’s proposal is ingenious. A supervaluation (over a model \( M \)) is a ‘valuation over classical valuations’; it is a function which assigns True to the sentences that are True in all classical valuations on \( M \); it assigns False to the sentences that are False in all classical valuations on \( M \); and no truth-value is assigned to the remaining sentences.

In this way, (1) and (2) are straightforwardly accommodated. With regard to (2), a supervaluation will always agree with classical valuations if the latter agree among themselves (which happens in the case of logical truths). But a supervaluation won’t be defined if classical valuations differ in their respective assignments (which happens in the case of sentences with non-denoting terms, for which there are different valuations). In this case, as required by (1), we have a truth-value gap.

According to Bencivenga (1986), a crucial difficulty faced by the method of supervaluations is that, if we try to extend it to the predicate case, it depends on certain
arbitrary conventions. In order to avoid these conventions, Bencivenga suggests a counterfactual theory of truth (Bencivenga, 1986, pp. 404–12). This theory agrees with the correspondence theory on the sentences which do not contain non-denoting terms; with regard those which contain such terms, the theory is formulated as follows: a sentence containing non-denoting singular terms is True (False) if it would be True (False) if these terms were denoting, no matter what their denotation were (Bencivenga, 1986, p. 404). But how can we cash out the notion of modality assumed in this account? At this point, the partial structures approach can be useful.

4. Putting the frameworks together

Let us first discuss how to combine the partial structures approach with free logic and supervaluations. We shall relate the semantics provided by the partial structures approach with that formulated for free logic. Perhaps it is the formal similarity between the two semantics which underlies Corcoran’s point (see the quotation of his review in the introduction above).

As we have noted, the partial structures semantics is characterised by a demarcation in the domain of interpretation: on the one hand, we have observable, actual objects, represented by $D_A$; on the other, we have unobservable, non-actual objects, represented by $D_O$. Moreover, the intersection of $D_A$ and $D_O$ is empty, and their union covers the whole domain of interpretation. The relationship between this feature of the partial structures semantics and the semantics for free logic should now be clear. As noted above, the crucial trait of the Meinongian picture is the division, in the domain of interpretation, between existing things and non-existing ones. In other words, on both semantics, a similar way of dividing the interpretation domain is put forward. As a result, non-referring terms are accommodated in the same way: they refer to non-actual objects.

The question then arises as to whether this demarcation in the interpretation domain commits one to non-existent objects. As we saw, the ‘fact’ that it does is taken as one of the major criticisms against the Meinongian picture (see Bencivenga, 1986). The commitment to non-existent objects seems to be too high a price to be paid to accommodate non-referring terms. Since the Russellian picture provides an account of these terms which doesn’t rely on non-actual objects, it is taken to be a more attractive proposal – at least at the ontological front. As we saw, in the Russellian picture, non-referring terms are really non-referring, and that’s why the interpretation function is partial; in the Meinongian approach such terms ultimately refer, but to non-existent objects. From a purely formal point of view, either picture provides the intended results (assuming certain conditions which we don’t need to examine here; see Bencivenga, 1986 and Lambert, 1991a). The difference between them arises at the ontological level: the Russellian picture is thoroughly actualist; the Meinongian isn’t.

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5 Of course, partial structures have their own logic (see da Costa et al., 1998). And one could explore the alternative of explicitly changing the underlying logic used in the partial structures approach to a free logic, and consider the formal and philosophical advantages of this move. (Roughly speaking, the constructions articulated in the partial structures approach are formulated in terms of classical logic. However, as argued in da Costa et al., 1998, the logic of quasi-truth itself is paraconsistent; it is a Jaskowski logic.) This is certainly an alternative to explore, and one of the motivations for this move derives from the connection – noticed by Corcoran – between pragmatic thinking, as represented in Mikenberg et al., 1986), and free logic.

6 The sense of ‘non-actual’ needs to be carefully considered here. On an anti-realist construal at least, we are not committed to the reality of such objects.
It turns out, however, that there is also a price to be paid by the Russellian picture: either (1) truth-value gaps are introduced (and bivalence is rejected), or (2) truth-values are arbitrarily assigned to statements containing non-referring terms (and we have an unmotivated semantics). With regard to (1), if a statement contains a non-referring term, it is natural to claim that it lacks truth-value. But then it will no longer be the case that every statement is either true or false (in other words, one has to relinquish bivalence). With regard to (2), in order to avoid gaps, one could assign a truth-value to the statement in question, but since the latter (by hypothesis) includes a non-referring term, there is nothing in the semantics which justifies which truth-value it should receive. Thus, the assignment of a value is arbitrary.

We have a dilemma here. On the one hand, the ontologically most attractive semantics for non-denoting terms (the Russellian picture) either seems to reject bivalence or seems to lead to arbitrary truth-value attributions; on the other hand, the Meinongian picture, which avoids both difficulties, seems to commit us to non-existent objects. Is there an alternative account of non-denoting terms which provides the best of both worlds? That is, is there a proposal which, without rejecting bivalence, leads neither to arbitrary assignments of truth-values nor to the commitment to non-existent objects? We think the answer is positive, provided that we move to the partial structures approach.

The main idea is that statements containing non-denoting terms do have truth-value, but they are best characterized as being quasi-true. We can conceive ways in which such statements would be true (for example, if there were objects in the relation described by them). However, in providing the (quasi-)truth-conditions for such statements, no ontological commitment to non-existent objects is required. As we saw above, we don’t have to assume the truth of the extensions of a partial structure. Thus, we don’t have to assume that there are non-existent objects to run the Meinongian strategy in terms of partial structures. In this way, the major charge against the Meinongian picture can be answered if we adopt the partial structures framework. This framework can thus be used to provide a semantics for free logic.

But how can we claim that the distinction between actual and non-actual objects in the domain of a partial structure doesn’t commit us ontologically to non-existing things? In other words, how can the partial structures framework mimic the Meinongian picture without incurring the same ontological commitments? The answer lies in the fact that with partial structures there is a change in the norm of cognitive discourse. Instead of claiming that our assertions about non-denoting terms are true, we propose to take them as quasi-true at best. What this means is that they do have a truth-value, but there is no commitment to their truth. And once truth is no longer the norm of cognitive discourse, there is no commitment to the entities entertained in that particular domain (in this case, non-existing objects).

We have noticed above that the notion of quasi-truth can be formulated independently of A-normal structures, and therefore we can claim that a sentence is quasi-true without invoking a full structure in which it is true (see Bueno and de Souza, 1996). Moreover, as we saw, truth is strictly stronger than quasi-truth. And this is why by claiming that statements with non-denoting terms are at best quasi-true, we can accommodate them without having to countenance non-existing objects.

The intuition underlying this move is familiar. A sentence \( \alpha \) is quasi-true if everything is as if it were true; in other words, if there were objects in the relations described by \( \alpha \), then \( \alpha \) would be true. What is important about this remark is that it provides straightaway a strategy to cash out the reformulation of supervaluations
provided by Bencivenga, the so-called counterfactual theory of truth (Bencivenga, 1986). In this reading, what a supervaluation does is to spell out the following counterfactual: a sentence containing non-denoting singular terms is True if it would be True if these terms were denoting, no matter what their denotation were.

One of the difficulties faced by Bencivenga’s approach is that it has to assume a primitive notion of modality to formulate the counterfactual account. Of course, the relevant counterfactual can’t be analysed in terms of possible worlds, since one of the motivations for the introduction of the counterfactual theory is to avoid ontological commitment to non-actual objects. In a language with non-denoting terms, what account of such terms would one provide? Moreover, as we noticed, Bencivenga’s approach was also devised as an alternative to some arbitrary conventions that he found in van Fraassen’s formulation of supervaluations, when the latter is extended to the quantificational case.

In terms of partial structures we can provide an alternative to the counterfactual theory which neither introduces worlds nor a primitive notion of modality. Modal operators can be formulated in terms of quasi-truth, and such operators can then be used to provide an account of the required counterfactual. The main idea is to explore the fact that a sentence $\alpha$ is quasi-true of everything is just as if it were true; i.e. our present epistemic situation doesn’t preclude that $\alpha$ is true, since there is a way of extending the partial information about the domain in question (i.e. there is an $A$-normal structure) in which $\alpha$ is true. Modal operators can then be introduced in the following way (see da Costa et al., 1998):

Necessarily $P$ is quasi-true if for all $A$-normal structures $P$ is true.
Possibly $P$ is quasi-true if for some $A$-normal structure $P$ is true.

In terms of this account of modality, the required counterfactual can be accommodated. What we need is to provide an account of a statement of the form ‘If it were that $\alpha$, then it would be that $\beta$’. Now, we say that a counterfactual $\alpha \rightarrow \beta$ is quasi-true (in a partial structure $A$) if $\beta$ is true in an $A$-normal structure $B$ in which $\alpha$ is true. Notice that this is indeed a counterfactual, since in this context we assume that the partial structure $A$ encompasses some information which is taken to be false (with respect to the actual world). What the auxiliary $A$-normal extension does is to extend the partial information in $A$ preserving as much information as possible from what is known to be quasi-true in $A$. As a result, if $\beta$ is true in an extension in which $\alpha$ is true, the resulting counterfactual will be true as well.

This account can be applied to accommodate the counterfactual version of the supervaluations approach. If no matter how we assign extensions to non-logical constants of the language in question, the resulting valuations agree on the resulting truth-values, we say that the supervaluation is true. The counterfactual content of this claim is clear: if all valuations were to agree (on the assignment of truth-values), then the supervaluation would be true (or false, depending on the case). However, by cashing out this counterfactual talk in terms of quasi-truth, in the way indicated above, we avoid having to introduce a primitive notion of modality and also no arbitrary conventions about the language have to be introduced. The various

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7 The literature on counterfactuals is, of course, huge (for references and further discussion, see Lewis, 1986, pp. 20-7). Our point here is only to indicate a way of accommodating the counterfactual talk involved in Bencivenga’s reformulation of the method of supervaluations.
possibilities are covered by the $A$-normal extensions. Thus, the basic intuitions underlying the method of supervaluations can be accommodated in terms of the partial structures formalism.\footnote{It might seem strange to put together the method of supervaluations and quasi-truth. After all, whereas the former allows truth-value gaps, the latter avoids them. However, one of the advantages of the combined framework is the fact that it allows us to retain bivalence, and also to take any \textit{supervaluational} gaps as an indication that the statement in question is best conceived as being quasi-true.}

5. Application

We shall close this paper by considering one application provided by the present framework. We shall consider how it handles the Liar paradox. As is well-known, the Liar states: \textquote{This sentence is False}. Now, assuming classical logic and its standard semantics, this immediately yields a contradiction: the Liar sentence is False and it is not-False. As van Fraassen (1968) has argued, supervaluations provide an account of the Liar. With the introduction of truth-value gaps, the supervaluation-theorist can avoid the inconsistency generated by the Liar sentence: the latter simply lacks truth-value. So one can\textquotesingle t derive the problematic conclusion that it is both False and not-False.

The most important reason to suppose that the Liar lacks truth-value is that, by introducing such gaps, one can avoid the paradox. It has been argued, however, that this approach to the Liar faces two important difficulties (see Priest, 1987). First of all, in its attempt to avoid the inconsistency at all costs, it assumes that any alternative approach to the paradox which accepts the inconsistency is inadequate. It goes without saying that this begs the question against the paraconsistent logician, who accepts the inconsistency, but resists the claim that it trivialises the system one is working with (see da Costa \textit{et al.} 1995). In other words, by adopting a convenient paraconsistent logic, the inconsistency is not threatening, since it doesn\textquotesingle t entail every sentence of the language in question.

Secondly, the introduction of truth-value gaps is unmotivated. There is no reason to suppose that the Liar lacks truth-value, other than to void the inconsistency. Moreover, there are no non-denoting terms in the Liar sentence. So, the Liar is a \textit{bona fide} meaningful sentence, and as such it seems \textit{ad hoc} to claim that it doesn\textquotesingle t have a truth-value. As opposed perhaps to the case of norms and conventions, which are typically taken not to be truth-apt, the Liar clearly can receive a truth-value. And the major argument which denies that, on the hands of the supervaluation theorist, begs the question against the paraconsistent logician. This raises the question: Is there an account of the Liar which avoids triviality without introducing truth-value gaps?

The framework suggested here provides an answer to this question. We noticed above that the logic associated with quasi-truth is paraconsistent (see da Costa \textit{et al.}, 1998). Thus, it allows one to accommodate the inconsistency generated by the Liar without triviality. From the fact that the Liar sentence is both True and not-True, it doesn\textquotesingle t follow that we can derive every sentence of the language. After all, with a paraconsistent logic, we are able to distinguish inconsistency from triviality (see also Priest, 1987). (Other semantic paradoxes can be similarly accommodated.)

But what happens if we formulate the Liar sentence in terms of quasi-truth? We have what can be called the \textit{quasi-Liar}: \textquote{This sentence is not quasi-true}. As is easy to see, the quasi-Liar is both quasi-true and not quasi-true. However, with the use of the
underlying paraconsistent logic, no triviality emerges (see Bueno, 1999c). In this way, by moving to the partial structures approach, one can accommodate semantic paradoxes, such as the Liar, without introducing triviality or putting forward truth-value gaps. The only claim is that the relevant statements should be taken as quasi-true.

As with so many other things, Corcoran was entirely right in indicating the connection between quasi-truth and free logic. As the remarks above indicate, there is indeed an important relationship between the two, as well as between quasi-truth and supervaluations. And if this connection is explored, as we have suggested here, a new combined framework emerges.9

References


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