CAN A PARACONSISTENT THEORIST BE A LOGICAL MONIST?*

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To Newton da Costa, teacher and friend

Abstract

In recent years, the debate about logical pluralism has been fierce, with challenging arguments being provided by both sides. According to the logical pluralist, there are several different logics, that is, several answers to the question ‘Is this argument valid?’ (see, e.g., da Costa [1997], and Beall and Restall [2000] and [2001]). On the other hand, according to the logical monist, there is only one logic, that is, only one right answer to the question about the validity of arguments (see, e.g., Priest [2000]). In this paper, I examine the impact of this debate on our understanding of paraconsistent logic. After putting forward a defense of logical pluralism – along the lines articulated by da Costa [1997] – I argue that the best stance for the paraconsistent logician is provided by logical pluralism. After all, paraconsistent theorists cannot make sense of their own practice in a logical monist setting.

INTRODUCTION

There was a time, a long time ago, when the question ‘What is the right logic?’ was not an issue. Had it been raised, it would have been answered quite simply: the right logic is the only one that exists! Similarly to the situation in geometry before the emergence of non-Euclidean geometries, logic was identified with the logical system...

* A number of the points put forward here emerged from discussions with Newton da Costa, who has always stressed the importance of adopting a pluralist view of logic, and who has articulated a comprehensive pluralist stance about the subject (see, in particular, da Costa [1997]). The paper is dedicated to him, for the incredible difference that meeting and working with him makes – a fact that all of his collaborators know so well. For illuminating discussions about the issues examined in this paper, my thanks go to him, as well as to JC Beall, Mark Colyvan, Steven French, Graham Priest and Scott Shafer. Thanks are also due to the helpful comments I received from three referees for this volume. Important changes were made as a result of their comments.
that existed at the time, and the choice between alternative logics didn't arise, simply because there was nothing there to choose from: there was only one logic.

The picture changed dramatically, of course, during the twentieth century, with the formulation of several logical systems: from extensions of classical logic (such as modal logic) to alternative and in some cases rival systems, such as paraconsistent logics, intuitionistic logics, and quantum logics (just to mention a few). Once there is a plurality of logics to consider, it becomes a substantial issue which of them (if any) is the right one.

In this paper, I will explore the implications of the plurality of logics to the debate over the nature of paraconsistency, examining in particular whether a paraconsistent logician can claim that there is only one right logic (that is, whether he or she can advocate logical monism). As I shall argue, there are several reasons why paraconsistency doesn't seem to be compatible with logical monism. I shall then indicate a version of logical pluralism that seems to be adequate to accommodate the nature of paraconsistent logic.

In talking about the nature of paraconsistent logic, I do not want to suggest that there is something peculiar (or peculiarly metaphysical) about this logic. I take the issue of the nature of a logic in the same way Duhem [1906/1954] took the issue of the nature of a physical theory. He characterized the nature of physical theories in terms of the role they played in scientific inquiry (the theories' aim), and the representational devices such theories provided to accommodate the phenomena (the theories' structure). Similarly, in the case of logic (and of paraconsistent logic in particular), its nature is characterized by the role the logic plays in drawing conclusions (the logic's aim), and the representational devices a logic provides to achieve this aim (its structure). What I am calling here the representational device of a logic has to do with the mechanisms provided by a logic to achieve its aim. For example, in the case of paraconsistent logic, its aim has always been to accommodate inconsistencies in conceptual systems. The relevant mechanisms provided by this logic have to do with those semantic and proof-theoretic features of a paraconsistent logical system that allow one to avoid triviality even in the presence of "contradictions".

As I shall indicate below, paraconsistent logic is one logic among many. The idea is that, depending on our aims in a particular area of inquiry, different representational devices are required, and so different logics should be put forward. This is part and parcel of the pluralist view advocated here. Following da Costa [1997], this formulation of pluralism stresses that a logic ultimately depends on the domain of inquiry under consideration. Different domains – such as quantum mechanics and constructive mathematics – may require different logics, and that's why there is no absolute answer to the question as to whether a given argument is valid or not.

1 LOGICAL PLURALISM AND LOGICAL MONISM

Two rival views about logic form the basis for the discussion that follows. According to logical pluralism, there are several logics that adequately characterize the notion of logical consequence. So for the logical pluralist, there is no absolute answer to the question "Does this sentence follow from those?" Any answer is always relative to a logic, and different logics provide adequate answers to such a question. On the other hand, according to logical monism, there is only one logic, and thus only one logic adequately instantiates the notion of consequence. In this way, for the logical monist the question about what follows from what has an absolute answer.

Given the way in which these two views have been formulated, it becomes clear that the notion of adequacy is crucial. But when do we say that a logic is adequate? What follows is not meant as a definition, but only as guiding idea to help us to avoid misunderstandings. Logic, as is now widely acknowledged, is basically the study of the relation of consequence. Given an argument, we typically have an informal idea as to whether it is valid or not. The adequacy of a logic depends on its capacity to justify as valid those inferences that are formally (or intuitively) taken to be so, and as invalid those that are not.

In this account, logic functions similarly to the way a scientific theory functions. Both logic and a scientific theory are put forward to accommodate certain types of phenomena: in the case of the former, the phenomena are the consequence relation in a given domain; in the case of the latter, observation reports obtained in a certain field of inquiry. Of course, new scientific theories often provide new descriptions and new ways of interpreting observation reports. For example, with the emergence of Newtonian mechanics, the scientific community no longer described a falling body in Aristotelian terms (that is, as a body moving toward its natural place); instead, the description of the body's movement was articulated in terms of gravity. Similarly, with the introduction of a new logic some inferences that have been initially taken as (intuitively) valid are no longer taken to be so. Thus, similarly to what happens in the case of a scientific theory, logic also plays a role in describing and reinterpreting the relevant phenomena.

But a scientific theory also helps us to shape and refine our intuitions about the physical world. Consider, for instance, the description of the movement of the sun relative to the earth in pre-Copernican physics, in contrast with how we describe this phenomenon now. Similarly, the adoption of new logics also helps us to shape and refine our judgments about logical consequence. For example, the emergence of free

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1 See Chiappini [1989] for a systematic account of Duhem's view.
logic helped to identify and assess existential presuppositions found in standard first-order logic, similar to the way in which the emergence of first-order logic helped to identify and assess corresponding existential presuppositions in Aristotelian logic (for details, see Lambert (ed.) [1991], and references quoted therein).

Given these remarks, we can then say that a logic is adequate if the informal (or intuitive) consequence relation presupposed in a given domain has the same features as the consequence relation provided by that logic. If a logic is adequate, it formally represents the consequence relation adopted in a particular domain; that is, the logic generates the consequence relation among propositions that is informally (intuitively) found in the domain in question.

Of course, we are comparing here an intuitive notion of consequence with a formal notion. So there is no way of actually proving that a given logic is adequate (in the technical sense of ‘proving’). The situation here is similar to the one found in the case of Church’s thesis, which also establishes a relationship between intuitive and formal notions. And as is well known, although Church’s thesis cannot be formally proved, an (informal) argument to support the thesis can be provided. Similarly, in the case of consequence, we also have to establish the relationship between an intuitive and a formal notion, and at best we can provide an (informal) argument to support the adequacy of a logic.

But the adequacy of a logic is not only a matter of the structural similarity between the informal consequence relation of a given domain and the formal counterpart provided by the logic. It is also a matter of the structural relationship between the objects and relations of the domain and the corresponding ordering of propositions yielded by the logic. In this sense, a logic should reflect the relations found among the objects of the domain in question. For example, quantum logic became such a fruitful tool to explore the quantum world because by studying the geometrical structure provided by this logic, we can determine relations between the propositions about quantum systems. In this sense, quantum logic is heuristically adequate for the quantum domain (see Putnam [1979]).

Similarly, one of the motivations for the introduction of paraconsistent logic was to avoid formulating ad hoc restrictions on the comprehension schema of set theory. In this way, the paraconsistent logician doesn’t have to try to avoid the generation of the well-known set-theoretic paradoxes by somehow restricting that schema (and then, given the restriction, finding some way to add further set-theoretic axioms so that classical mathematics can be obtained). Instead of this, with the emergence of paraconsistent logic, a new way of investigating the foundations of set theory was devised. Paraconsistent logic allows one to study certain formulations of set theory in which the paradoxes are not avoided. But despite the inconsistency of the resulting theory, the latter is by no means trivial — or, at least, it is trivial if and only if classical (axiomatic) set theory is inconsistent (see da Costa [1986], da Costa [1997], da Costa, Béziau and Bueno [1998], and Weir [1998]). In this way, paraconsistent logic is heuristically adequate for the investigation of inconsistent domains: it allows one to explore the inconsistent without triviality.

It might be argued that this is not a role that we should expect a logic to play. What has to be heuristically fruitful (or adequate) is a theory about the domain in question, not a logic. A logic is only concerned with the study of consequence relations in a language; heuristics has no part in this.

In reply, note that a theory typically presupposes the existence of a logic. In the so-called syntactic formulation, a theory is conceived as a set of sentences closed under logical consequence. Even if a theory is presented, following the semantic approach, as a family of models, a particular logic is assumed in the formulation of these models. (Models are not “free floating” entities, but their features depend on the theory in which they are expressed — typically set theory — which in turn presupposes a logic.) So to claim that heuristic fruitfulness is a property of a theory, and not of a logic, is to disregard a crucial feature of a theory: its dependence on logic. And depending on the logic one uses, different consequences are obtained, and so different theories are formulated. Thus, we can say that logic has a crucial role in the representation of the domain to which it is applied, in the sense that the structures that are generated by the logic in question reflect the relations among the objects in the domain (the examples of quantum logic and paraconsistent logic clearly illustrate this).

But if there are several consequence relations (that is, several answers to the question “What arguments are valid?”), the issue arises as to how we can choose between the resulting logics. Depending on the particular philosophical attitude one advocates toward logic, different answers are forthcoming. On the one hand, there are those that claim that a logic should be true to be acceptable. According to this realist proposal, truth provides the crucial criterion to decide which logic one should adopt. The main idea is that logic deals with truth-preserving methods, and that a logical system is true if it completely captures the consequence relation employed in a given domain. Realists will use various (partial) criteria to assess whether a logic is true or not. Usually the outcome of such assessments is tentative, but it is enough for realists to provide an account of logic. On the other hand, there are those who do not advocate truth, but rather a weaker aim for logic. For example, extending to logic Field’s [1980] anti-realist proposal for mathematics, anti-realists can say that in order for a logic to be good it only has to be conservative over atomic statements (see Akiba [2000]). According to this proposal, no new consequence about atomic statements can be derived by adding a logic to a body of atomic statements. Given that in this account a logic doesn’t have to be true to be good, this proposal provides a form of anti-realism about logic. And given that anti-realists do not take truth as an

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1 For two quite different versions of realism about logic, see Popper [1963] and Priest [1987].
can a Paraconsistent Theorist Be a Logical Monist?

comes as no surprise that we often find logical monism defended by realists, whereas anti-realists so often put forward logical pluralism.

2 THREE FORMS OF LOGICAL PLURALISM

Logical pluralism comes in different versions. A highly original development and defense of logical pluralism was provided by da Costa, highlighting how a logic depends on the domains to which it is applied (see da Costa [1997]). On the other hand, Beall and Restall provided a systematic and thoughtful defense of logical pluralism in terms of the notion of cases (see Beall and Restall [2000] and [2001]). The main difference between these two versions of logical pluralism is that da Costa highlights the role played by domains in the selection of a logic, whereas Beall and Restall emphasize the role played by cases in the formulation of a logic. The difference is not verbal, though. As conceived by Beall and Restall, a case indicates the particular metaphysical assumptions assumed by a given (formulation of) logic. In fact, according to the intuitive notion of consequence, \( \alpha \) is a consequence of \( \Gamma \) if there is no case in which each premise in \( \Gamma \) is true, but in which \( \alpha \) is not true. As Beall and Restall point out, cases range from (set-theoretic) models and possible worlds to situations (but are not restricted to these things). And depending on the nature of such cases, different answers to the question of which arguments are valid emerge. The crucial claim that makes Beall and Restall’s position a version of pluralism is that, in their view, there are at least two different specifications of cases that satisfy the intuitive notion of consequence, and so there is more than one answer to the question about the validity of an argument.

Da Costa’s domains, in turn, are employed primarily as a way of selecting the adequacy of a logic, indicating the domain to which it applies. In selecting which logic to apply we may consider particular features of the domain in question. For example, if our aim is to examine constructive features of mathematical reasoning, intuitionistic logic is far more adequate than classical logic; if our aim is to study the features of an inconsistent domain, paraconsistent logic fares systematically better than classical logic. Domains are then crucial in logic selection.

This is not the place, however, to compare these two versions of logical pluralism (I develop such a comparison in Bueno [2001]). And my argument in the present paper doesn’t actually depend on the particular version one adopts. For simplicity, I will follow da Costa’s version.

Independently of the version of pluralism one considers, there are three forms of logical pluralism.\(^4\) (i) The most common type is pluralism about pure logic (see da

\(^3\) In those cases in which realists and anti-realists agree about which criteria to adopt (for example, simplicity and unity), they do not assign the same significance to their satisfaction. Realists typically take such criteria to be epistemic; anti-realists take them as pragmatic at best (see van Fraassen [1980] and [1985]). I will return to this distinction below.

\(^4\) Priest [2001] identifies and briefly examines the main features of these three forms of logical pluralism (his criticism focuses on the third version). As will emerge below, I don’t think that Priest’s criticism succeeds.
Costa [1997], and also Priest [2001]). We can call this type of pluralism "pure logical pluralism". Pluralism here amounts to the (undeniable) fact that there are several pure logics; that is, several purely formal structures that characterize logical consequence. The contrast between pure and applied logic can be clearly made in comparison with the similar contrast between pure and applied geometry. As da Costa [1997] points out, we can consider geometry from two different viewpoints: applied geometry (as the use of geometrical systems to describe physical space). Similarly, in the case of logic, there are pure logics (as the abstract study of consequence relations) and applied logics (as the use of such systems to describe different domains, from electronic circuits to grammatical structures). Pure logical pluralism is undeniable and uncontroversial. It is a mathematical fact that several different pure logics have been developed and are currently being studied.

(ii) The second level of logical pluralism focuses on theoretical applications of logic. (Priest [2001] calls this level of pluralism "theoretical pluralism"). According to this proposal, logics can be applied to several domains. So this form of pluralism focuses on the application of logic, rather than on its pure formulation. The domains of application range from grammatical structures through quantum systems to mathematical fields. According to the theoretical pluralist, typically there are several logics that can be applied to a given domain — although a given logic is usually more adequate to certain domains than to others. For example, both classical logic and paraconsistent logic can be applied to consistent domains, and typically they will generate the same results there. However, once we move to inconsistent domains, classical logic is no longer an adequate option (unless we want to allow the arbitrary rejection of some bits of information about such domains). After all, by identifying inconsistency and triviality, classical logic precludes the possibility that, in inconsistent domains, we fail to validly infer at least one sentence of the language we use. On the other hand, paraconsistent logic allows inconsistency to be accommodated, since this logic doesn't arbitrarily require one to abandon some bits of information about the inconsistent domain, and so inconsistency need not be avoided at all costs. Of course, we still need to avoid triviality. But if we have a logic that distinguishes inconsistency from triviality, the avoidance of triviality provides no reason to avoid inconsistency.

Finally, (iii) there is a version of logical pluralism that concentrates on the canonical application of logic, namely to reasoning in natural language (reasoning in the vernacular). For lack of a better term, this version of pluralism can be called "canoncal pluralism". Canonical pluralism is a particular case of theoretical pluralism, when the application considered is the canonical one. According to the logical pluralist, even in the case of the canonical application, more than one logic are adequate to the domain under consideration. The plurality of logics goes "all the way down".

According to Priest [2001], only canonical pluralism provides an interesting case for logical pluralism. After all, pure logical pluralism and theoretical pluralism only claim that there are different logics — an undeniable, but ultimately uninteresting, mathematical fact. The only kind of pluralism that Priest takes to be challenging is canonical pluralism, and it is on this kind of pluralism that he concentrates his case. Priest takes it that theoretical pluralism and pure logical pluralism are not substantial enough.

But is this assessment correct?

3 LOGICAL PLURALISM DEFENDED

I beg to differ. As opposed to Priest's suggestion, I don't think that pure logical pluralism is a trivial (i.e. uninteresting) proposal. On the contrary, the fact that pure logical pluralism is a recent phenomenon indicates that its emergence is anything but trivial. The idea that there are several distinct (pure) logics — the idea that one could study several different bona fide logical systems — is very recent; it was only in the twentieth century that the idea was seriously entertained. This indicates that it is a substantial issue whether one can explore different logical systems in the way suggested by the pure logical pluralist. That we can make such explorations is a fact; but not a trivial one.

As a sociological claim about logic, Priest's claim that "pure logical pluralism is trivial" is not contentious. It is a fact that currently there is a huge variety of pure logics around. But this is not the claim that the logical pluralist is concerned with. Logical pluralism is a philosophical claim about logic; it's not a sociological view. And from a philosophical point of view, it is far from trivial that a plurality of pure logics can legitimately be developed. Similarly to what has happened with geometry, it might be thought that it would be incoherent to develop alternative logical systems. It might be thought that such systems would never be consistent. Or it might be claimed that alternative pure logical systems are not even logic: such systems simply change the subject. (This is a reaction that we still find today, even since Quine [1970] introduced the argument.) The fact that such systems can be developed, the fact that such systems can legitimately be considered logics, and the fact that there are substantial disagreements about this issue indicate that the
plurality of pure logical systems is far from trivial. The question that the pure logical pluralist raises is clear: what should one make of this plurality? But the answer to the question is anything but trivial.

I also take it that there is a substantial issue as to whether theoretical pluralism is trivially true or not. Priest [2001] suggests that theoretical pluralism does not raise a serious problem for the logical monist. After all, he argues, the monist can always adopt the usual criteria of theoretical evaluation to choose between rival applied logics (such as simplicity, unity, no ad hocness, adequacy to the data etc.). But there are two difficulties with this suggestion: (1) some of these criteria (in particular, simplicity, unity and no ad hocness) only provide pragmatic reasons to accept a theory, and such reasons typically are not epistemic. A pragmatic reason indicates the usefulness of adopting a given theory (or applied logic). It is easier for us to work with theories and logics that are simple and unified. But why does this provide any reason for us to believe that such theories and logics are true? As opposed to pragmatic reasons, epistemic reasons are those that evaluate the relationship between the theory and the world, and so increase the chance that a given theory or logic is true. But simplicity, unity and no ad hocness on their own are not epistemic in this sense (see van Fraassen [1980] and [1985]). (2) Moreover, even if those criteria were epistemic, they typically do not uniquely select one theory in a given domain. After all, usually more than one theory satisfies the epistemic criteria. So we still end up having more than one theory underdetermined by the data. In other words, even with those criteria, theoretical pluralism, rather than monism, emerges.

The point of these remarks is to indicate that one typically cannot overcome a pluralist view about logic simply by suggesting that there are criteria of logic selection, anymore than one can overcome a pluralist view about science by mentioning that there are criteria of theory selection. In both cases, one has to show that the criteria uniquely determine the outcome. Otherwise, a pluralist view would still be justified. The fact that in both cases (in science and in applied logic) there is underdetermination of theories/logics by the data raises a serious problem for the (scientific and logical) monist. If there are more than one theory (or more than one logic) that is adequate for the domain in question, the monist view cannot be correct. (This is the point of (2) above.) And even if the criteria could uniquely select one theory (or one logic), given that the criteria are mostly pragmatic, they do not establish that the selected theory or logic is true — or, at least, that we have reason to believe that they are true. (This is the point of (1) above.)

In other words, given the underdetermination of theories and logics by the data, there will typically be more than one logic adequate to a given domain. Even if we were to use other criteria for theory/logic selection, typically there wouldn’t be epistemic reasons for preferring a theory or a logic over another. And if the reasons in question are pragmatic at best, they fail to establish that the selected logic is true.

In other words, one should support logical and theoretical pluralism rather than their monist counterparts.

For these reasons I don’t think that pure logical pluralism and theoretical pluralism are trivial. It is important to realize this point, since Priest often uses the strategy of turning the claims made by the logical pluralist into claims about pure logical pluralism or about theoretical pluralism, and then he just remarks that these claims are trivial. They are not. And as I tried to indicate above, one cannot claim that the logical pluralist’s claims are trivial in this sense. What about canonical pluralism then? This is the only version of pluralism that Priest thinks is interesting, and I will discuss now the main argument against this form of pluralism that he provides (see Priest [2001]).

According to Priest, the main worry that the canonical pluralist faces is that his or her position is simply incoherent. Let us grant that logic depends on the domain under consideration as the pluralist claims. Why should we think that the different logics that are adequate to the domain in question are actually rival (with respect to the inferences under consideration)? Well, either these logics are rival or they are not. If they are not rival, there is no reason to support a pluralist view. After all, if there is no disagreement between the logics (in the sense that they actually generate the same set of consequences), the pluralist will have to acknowledge that there is no reason to distinguish them, no reason to choose one rather than the other. In this case, for all practical purposes, we have only one logic — in complete agreement with logical monism. On the other hand, if the logics under consideration are rival, then only one of them (if any) will be right about the representation of the inference in question. But in this case, once again, logical pluralism fails to be a viable option, since the conclusion supports logical monism! So in either case the logical pluralist doesn’t provide an adequate response to the issue. To the extent that a response is discernible at all, it seems to conflict with the main features of logical pluralism.

According to Priest, this line of argument actually provides evidence for logical monism. And it certainly seems to raise a serious difficulty for the logical pluralist. But I think the difficulty is only apparent. Priest has introduced a false dilemma. For the logical pluralist, it is simply not the case that if two logics (applied to the same domain) are rival, then only one of them is right. Only a logical monist would grant that. The fact that there are logics that are inadequate for the domain in question doesn’t entail that only one logic is adequate. More than one logic may be adequate — which is exactly what the logical pluralist claims! The logics may provide the same results with regard to inferences in the common domain (and that is why they are adequate with respect to that domain), although they may provide different results with regard to inferences beyond the domain (they are different logics, after all). The adequacy of a logic, as we saw above, depends on the aim and structure of the logic in question. Two different paraconsistent logics may provide the same results with respect to a given inconsistent domain, even though they may differ in
other domains: one logic may be trivialized by a "contradiction" that doesn't trivialize the other (and both logics will agree with classical logic in a consistent domain).

Priest also criticizes da Costa [1997] for putting forward a pluralism that is domain-dependent; a pluralism according to which depending on the domain we consider, different logics are adequate. For example, a constructivist logic is adequate for domains that involve constructive inferences; and a Schrödinger logic is adequate to the quantum domain. According to Priest, this form of pluralism cannot make sense of the fact that sometimes we need to use a logic in overlapping domains. For example, we may need to study constructive inferences in the quantum domain.

I don't think this raises any difficulty for the domain-oriented pluralism of da Costa. After all, we can simply combine both logics (constructivist and Schrödinger logics) to accommodate the overlapping domain in question. And note that this is exactly what the logical pluralist should say. After all, when we have to consider the overlapping domain (in which both logics hold), strictly speaking, we have changed the domain. A different aim has to be achieved. Instead of only having to accommodate constructive inferences or having to accommodate inferences in quantum mechanics, the aim has changed to accommodate constructive inferences in quantum mechanics. And to achieve this aim a different structure is required: one that provides a constructive consequence relation defined for objects that may not have identity conditions. Once we realize this, it then becomes clear that more than one logic meets this requirement: several constructivist logics and several Schrödinger logics can be used to obtain a logic for the overlapping domain, with the result that more than one logic will be adequate to such a domain. For these reasons I think Priest failed to establish that logical monism should be the right answer to the question about what inferences are valid.

4 LOGICAL PLURALISM AND PARACONSISTENCY

Once we have seen that logical pluralism is a viable option, the question arises as to what attitude the paraconsistent logician should adopt about this issue. In other words, what is the status of paraconsistency in the logical pluralism debate?

It will come as no surprise now that I think the paraconsistent logician should be a pluralist. There are a number of reasons for this claim. First, there are different kinds of "contradictions", depending on the domain of knowledge we consider, and there are different ways of dealing with them – different types of paraconsistent logics that are adequate to accommodate the inconsistencies in question. So in order to maintain a monist view, the paraconsistent logician would have to specify criteria of selection of alternative paraconsistent logics. But how can that be done?

Consider, for example, the first paraconsistent system that incorporated both the propositional and the predicate calculus (among other features): da Costa’s $C_n$ logics (for an overview, see da Costa [1974]; see also da Costa, Béziau and Bueno [1995]). This system provides a hierarchy of paraconsistent logics $C_n$ ($1 \leq n \leq 6$), that in a certain sense encompasses classical logic, and is such that each logic in the hierarchy is strictly weaker than the previous one (and so a "contradiction" that trivializes $C_1$ doesn’t trivialize $C_{11}$). So there is not simply one paraconsistent logic but infinitely many. Which of them is the right one?

Of course, as a pure piece of formalism – as a pure logic, as da Costa [1997] would say – there is no issue of a logic being right or wrong (or being adequate or inadequate). At best there are constraints on the acceptability of the formalism, e.g. whether the formalism is sound, complete, decidable etc. But these constraints have nothing to do with the logic being right or wrong; rather they indicate whether the logic is in question is formally acceptable. It goes without saying that there are substantial disagreements about the importance of these formal constraints. For example, in the debate between those that claim that second-order logic is not really logic and those that argue that it is, the first-order theorist takes the completeness of a logical system as a necessary condition for its acceptability. The second-order theorist disagrees of course (see Shapiro [1991]).

It is typically in the context of the application of a logic that the issue of its adequacy emerges. As an applied logic, which of the infinitely many paraconsistent logics is the right one? The answer depends, of course, on the kind of inconsistency that we are dealing with. As noted above, there are some "contradictions" that trivialize even a paraconsistent system. For example, even in paraconsistent arithmetic, if it is established that $1 = 0$, the resulting system is trivialized. Moreover, in da Costa’s $C_n$ logics, there are "contradictions" that trivialize certain logics $C_n$, although they may not trivialize the logic $C_{11}$. Despite these facts, there still are several paraconsistent logics adequate to accommodate inconsistent domains. And so although not every paraconsistent logic is equally adequate for a given inconsistent domain, more than one are.

As a result, it is difficult to see how paraconsistent logicians can make sense of their own practice if they were to adopt logical monism. Not only should the

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1 According to some interpretations of quantum theory, identity cannot be applied to quantum particles. To formally accommodate this situation, da Costa and Krause introduced Schrödinger logics (for details, see da Costa and Krause [1994] and [1997]).

2 Classical logic is preserved in the sense that the principles of classical positive logic are valid in paraconsistent logic. Moreover, it is possible to introduce a negation in the paraconsistent system that behaves exactly like classical negation (see da Costa [1974]).

3 Da Costa [1997] makes this point, and so does Priest [2001].
paracommon logician acknowledge the plurality of paraconsistent logics (both pure and applied), but also in consistent contexts, classical logic is contained in paraconsistent logic, and so in such contexts both logics are adequate. This is the case even of Priest’s LP, which, as opposed to da Costa’s $C_n$ logics, is only one logic rather than a hierarchy of logics (for details, see Priest [1987]). In other words, if Priest claimed that the one true logic is LP, he still needs to acknowledge (as he does) that in consistent contexts LP and classical logic yield the same results. What this means is that such logics are equally adequate to deal with consistent domains. And so any commitment to one logic (in a consistent domain) will be extended to a commitment to the other logic (in such a domain). Paraconsistent logicians that advocate logical monism cannot deny this; if they do, they will end up being incoherent, given that they would be denying the very logic they advocate — after all, classical and paraconsistent logics yield exactly the same results in consistent domains. And this kind of incoherence is problematic — even for the paraconsistent logician!

The above argument is perfectly general, and it applies to whatever paraconsistent logic one considers: the $C_n$ logics, LP, Jaskowski’s logic (see D’Ottaviano and da Costa [1970], and da Costa, Bueno and French [1998]), LFI (see Carnielli, Marcos and de Amo [2001]), and so on. To the extent that in consistent domains paraconsistent logics agree with classical logic, a substantial form of pluralism will immediately emerge. And this is a pluralism that the paraconsistent logician cannot coherently deny. In other words, it is difficult to see how a paraconsistent logician can be a logical monist. Given that in a consistent context, classical logic and paraconsistent logic yield the same results, the paraconsistent logician needs to acknowledge the adequacy of classical logic in consistent domains.

Moreover, logical pluralism provides an adequate stance to make sense of non-classical logics in general. After all, there are close connections between classical logic and its non-classical counterparts. For example, in finite domains even the intuitionist logician recognizes that the excluded middle applies. And in domains involving macro-objects, the quantum logician recognizes that classical logic applies; not to mention that in order to provide a semantics for quantum logic, typically the quantum logician relies on classical logic. Once pure logical pluralism and theoretical pluralism are considered live and interesting options — as they should be — the paraconsistent logician cannot coherently deny them.10

Let me elaborate on this. I’ve claimed that the quantum logician needs to be pluralist about logic, given that the semantics of quantum logic relies on classical logic. This is because the semantics for quantum logic is given in classical set theory, and the latter presupposes classical logic. Of course, for the quantum logician to provide a semantics for quantum logic without relying on classical mathematics, he or she has to develop a quantum set theory. But even if we grant that there are excellent motivations for the development of quantum logic on the basis of quantum mechanical evidence — such as the failure of the distributive law in the outcome of certain spin measurements — there is no corresponding failure in set theory per se. Thus some independent evidence for the development of quantum set theory seems to be needed.11

The situation is quite different in the case of paraconsistent logic. There are independent reasons to develop a paraconsistent set theory, besides the need for providing a semantics for paraconsistent logic (as is not based on classical set theory). For example, with a paraconsistent set theory we can study genuinely inconsistent structures — such as the Russell set and the original version of the infinitesimal calculus — that cannot be properly studied otherwise. In other words, those structures cannot be studied as inconsistent structures without a paraconsistent set-theoretic framework (for details, see da Costa, Béziau and Bueno [1998]). The important point here is that, depending on the particular paraconsistent logic one adopts, different paraconsistent set theories emerge (see, again, da Costa, Béziau and Bueno [1998]). This provides a strong case for theoretical pluralism, since more than one paraconsistent logic can be used to articulate a paraconsistent set theory. Once again, this is a significant form of theoretical pluralism that a paraconsistent logician cannot coherently deny.

In this way, it becomes clear that logical pluralism provides an adequate framework for the paraconsistent logician to understand and articulate his or her own practice. It also provides a framework in which paraconsistency can flourish, given the integration it allows of the aim and structure of paraconsistency, and the search for a unified approach to study inconsistent domains.

5 CONCLUSION

The answer to the question posed in the title — “Can a paraconsistent theorist be a logical monist?” — should now be clear: No. Given the arguments above, it is difficult to see how a logical monist can make sense of certain systems of paraconsistent logic (and, of course, the whole point of advancing a philosophy of logic is to make sense of logic!). Da Costa’s $C_n$ logics, as we have seen, with its hierarchy of paraconsistent logics, immediately raises the issue of pluralism. In this

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10 It comes as no surprise that, in the process of defending logical monism, Priest denies that these two versions of pluralism (pure logical and theoretical pluralism) are relevant. But as I argued above, I don’t think his assessment is right.

11 I am not suggesting that quantum set theory should not be developed as a piece of pure mathematics. The question is whether we have motivation to develop such a set theory on the basis of applied mathematics — beyond the need for providing a semantics for quantum logic, of course.
system, there is not simply one paraconsistent logic but infinitely many. Which of them is the right logic? The logical monist has to provide an answer to this question, by spelling out criteria to decide what is the right paraconsistent logic. But, as we saw, there are substantial difficulties to achieve that. In the end, the best account of the plurality of logical systems is still provided by logical pluralism: there are several different logics each of them adequate to certain domains. Thus logical pluralism is still the best stance for the paraconsistent theorist – the best stance to make sense of paraconsistency.

REFERENCES


