PARACONSISTENCY:
TOWARDS A TENTATIVE INTERPRETATION

Newton C.A. DA COSTA*
Otávio BUENO**

Manuscript received: 1998.9.13.
Final version: 1999.10.15.

* Department of Philosophy, University of São Paulo, P.O. Box 8105, São Paulo-SP, 05508-900, Brazil. E-mail: nca_costa@usp.br
** Department of Philosophy, California State University, Fresno, CA 93740-0105, USA. E-mail: otavio_bueno@csufresno.edu

BIBLID [0495-4548 (2001) 16: 40; p. 119-145]

ABSTRACT: In this expository paper, we examine some philosophical and technical issues brought by paraconsistency (such as, motivations for developing a paraconsistent logic, the nature of this logic, and its application to set theory). We also suggest a way of accommodating these issues by considering some problems in the philosophy of logic from a new perspective.

Keywords: paraconsistent logic, paraconsistency, paraconsistent set theory.

CONTENTS

Introduction
1. Logic, mathematics, paraconsistency
2. Motivation: paraconsistency and set theory
3. Paraconsistency: remarks on its theoretical status
   3.1. Pure logic, applied logic and paraconsistency
   3.2. Logic, quantum mechanics and paraconsistency
4. Paraconsistency: some technical applications
   4.1. Russell’s set and paraconsistency
   4.2. A paraconsistent Boolean algebra
   4.3. Semantic analysis of a paraconsistent logic
5. A case study: syllogism and paraconsistency
   5.1. Classical syllogistic
   5.2. Paraconsistent syllogistic
6. Concluding remarks
Bibliography

The mathematician will have to take account not only of those theories that come near to reality but also, as in geometry, of all logically possible theories, and he must always be careful to obtain a complete survey of the consequences implied by the system of axioms laid down.

David Hilbert: 1901, 'Mathematical Problems.'
The models we used in the previous study were based on classical linguistic and cognitive theories, but these models have been expanded and refined to include more recent developments in the field. Our new model takes into account various factors, including the interaction between the model and the user, the context in which the model is used, and the specific tasks that the model is designed to perform. This new model is more flexible and can be applied in a wider range of situations. In addition, it is more accurate and can provide more precise results than the previous model. Overall, the new model is a significant improvement over the old one and will be useful for a variety of applications.
Gödel's theory of constructive sets fall within the general theory of models, and those who are not acquainted with such themes do not have a notion of the current state of the evolution of logic. If we recall that classical model theory has already given us several topics that one cannot disregard (prime models, saturated models, categoricity in potency and Morley's theorem, omission of types, elimination of quantifiers, real closed fields, classification theory (Shelah) etc.), one immediately notices the enormous richness of general model theory or mathematical semantics. (For further comments on semantics, see da Costa, Bueno and Béziau 1995.)

The theory of machines, or recursion theory, has been so developed in the last years that it is no longer possible to follow its literature in all of its details. To the traditional themes such as, recursive functions, Rice's theorem, arithmetical hierarchy, analytical hierarchy, Post languages etc., many others have been added, making this area of logic still richer. Some have tried, for instance, to extend the notion of calculability through Turing machines or recursive functions, as is the case of Smale and his collaborators.

Everything that was just recollected supplies evidence for the fact that the development of pure logic is the same as that of pure mathematics. Understanding its nature and meaning is equivalent to understanding, in general, the meaning and nature of pure mathematics. It is enough to notice here that its progress, at least in principle, is made on an a priori and abstract level; experience (thought of in a comprehensive sense), both related to common life as well as to science, has only a heuristic value.

Regarding applied logic, just as with applied mathematics, things are quite distinct. Logic, for instance, thought of as the science of the valid forms of inference, is placed within this domain, that is, within applied logic. The problem, in this case, consists in discovering abstract structures that reflect the real mechanisms of deductive inferences in a certain domain. Thus, one can be concerned with inferences found in ordinary life, in both traditional and constructive mathematics, as well as in quantum mechanics and natural sciences.

As opposed to its pure counterpart, applied logic is not articulated in an abstract and a priori level. On the contrary, it somehow depends on experience (in a comprehensive sense) and on pragmatic factors as well (theoretical simplicity, intuitiveness, capacity of systematisation and so on). For this reason, the constructive study of constructive mathematical thought does not fit with classical logic schemes; in other words, the categories and processes of classical logic (principle of excluded middle, classical method of reductio ad absurdum etc.) cannot reflect the mechanisms underlying constructive thinking. Hence the existence of various constructive logics (Brouwer-Heyting, Griss,...).

Analogously, standard quantum mechanics (as it will be argued in section 3.2) seems to lead to non-classical logics, provided that one wishes to account adequately for what happens at the quantum domain (modular logics, orthomodu-}

logics, Kochen-Specker structures (see Kochen and Specker 1967 etc.).

The numerous, but interconnected, topics presented in this paper were chosen in order to clarify some aspects of the nature of paraconsistent logic, its meaning and significance. It should be clear from the outset that everything that is said concerning logic in general obviously also holds for paraconsistent logic. In particular, the latter can be viewed both as a pure subject as well as an applied one. In the first case, just as the rest of mathematics itself, it is concerned with conceptual structures defined and investigated in an a priori way. In the second case, thought of as an applied discipline, it depends on experience and is dependent on pragmatic constraints.

The division between pure and applied logic within a paraconsistent domain is extremely important, allowing in particular the better examination of some problems. As we shall see in section 3.1, some specialists criticise certain paraconsistent systems for the fact that in these systems the law of substitution of equivalents does not hold; but they prefer to say that this is a valid law. However, from the perspective of pure logic, such a critique would be similar to that made by an algebraist who wishes that only commutative groups be studied... From the applied standpoint, nevertheless, such a discussion might be relevant, provided that one is taking into account certain applications, for instance, to the domain of computer science. Unfortunately, though, this is frequently not what happens, being just as if one has access to a platonic, true logic, adopted as a standard of comparison between all the alternative logical systems under consideration. On the contrary, when one is concerned with an issue of applied paraconsistent logic (for instance, to expert systems), it makes sense to determine whether a certain property, such as the one just mentioned, is or is not to be met by the logical system being outlined. Furthermore, it is possible to ask whether in some expert systems, in order to handle contradictory bits of information, it is appropriate that the underlying logic, of a paraconsistent kind, has a second negation, which behaves classically.

Having presented these general remarks concerning logic, mathematics and paraconsistency, before considering, in section 3, some aspects of the theoretical status of the latter, we shall briefly examine some of the main motivations for its introduction.
The following formula (or scheme of formulation)

$$
\neg \phi \lor \neg \psi
$$

becomes the following formula (or scheme of formulation)

$$
\phi \land \psi
$$

Let the set of all propositions be divided into two groups: the set of all propositions that are true and the set of all propositions that are false. The members of these two groups are called truth-values, or truth-values of propositions. The truth-value of a proposition is determined by the truth-value of its constituent propositions. The truth-value of a compound proposition is determined by the truth-values of its constituent propositions and the logical connective that combines them.

In formal logic, propositions are grouped into sets, or classes, based on their truth-values. The set of all true propositions is called the class of all true propositions, and the set of all false propositions is called the class of all false propositions. The logical connectives and quantifiers are used to combine propositions into larger propositions.

The logical connectives are:

- Conjunction (\(\land\))
- Disjunction (\(\lor\))
- Negation (\(\neg\))
- Implication (\(\rightarrow\))
- Equivalence (\(\iff\))

The quantifiers are:

- Universal Quantifier (\(\forall\))
- Existential Quantifier (\(\exists\))

In a formal system, the truth-value of a proposition is determined by the truth-values of its constituent propositions and the logical connectives that combine them. The truth-value of a proposition is determined by the truth-values of its constituent propositions and the logical connectives that combine them. The truth-value of a proposition is determined by the truth-values of its constituent propositions and the logical connectives that combine them.

In formal logic, the truth-value of a proposition is determined by the truth-values of its constituent propositions and the logical connectives that combine them. The truth-value of a proposition is determined by the truth-values of its constituent propositions and the logical connectives that combine them. The truth-value of a proposition is determined by the truth-values of its constituent propositions and the logical connectives that combine them.
clearer understanding of negation; the possibility of the discourse, even if one partially rejects the principle of non-contradiction; a proof that this principle is at least partially true, and so on.

It would be natural to think that, being based on somewhat different motivations and presenting distinct features, paraconsistent logic and the classical one might somehow dissent as far as their theoretical status are concerned. Things however may not be this way - an issue to which we now turn.

3. Paraconsistency: remarks on its theoretical status

3.1. Pure logic, applied logic and paraconsistency

Logic is usually considered as an *a priori* and analytic domain; it is taken to be independent of experience, and its laws are thought of as compatible with any contingent state of affairs that might happen. This view, however disseminated, is by no means undisputed; indeed, as Heisenberg stressed a long time ago:

(...) if one wishes to speak about the atomic particles themselves, one must either use the mathematical scheme as the only supplement to natural language or one must combine it with a language that makes use of a modified logic or of no well-defined logic at all (Heisenberg 1958, p. 46).

And Schrödinger has also noticed:

As our mental eye penetrates into smaller and smaller distances and shorter and shorter times, we find nature behaving so entirely differently from what we observe in visible and palpable bodies of our surrounding that no model shaped after our large-scale experiences can ever be true (Schrödinger 1952).

Both remarks are symptomatic of a striking fact: quantum mechanics unavoidably leads to logical settings distinct from the classical ones. As far as we know, and as we shall argue for in the next section, it seems that there is a quantum logic considerably diverse to that found in our traditional logical framework. Nevertheless, as is well known, all the argumentation concerning the logical foundations of quantum physics is not developed in *a priori* lines; instead, experiments, such as Gerlach's and Stern's on the spin of particles, as well as quantum laws, such as Heisenberg's principle, should be taken into account - and these are the experiences and laws that made us reconsider the basis of the underlying logic of physics.

Intuitionistic logic, for its part, is one of the possible adequate ways employed in order to systematise constructive thinking in mathematics. Classical logic by no means reflects the constructive activity of the mathematicians, for it depends on the implicit assumption that they work in domains composed by objects already given, whose existence their constructive work is not concerned with.

Thus, quantum and intuitionistic logics supply evidence for the thesis that logic, in its applications, is dependent on the particular features of the domain that it organises. It is plain that we are referring here to applied logical systems, and not to pure logic. The pure logician, of course, can elaborate and scrutinise any system, independently from the experience. However, regarding their applications, there is the inter-connection between the logical dimension and the domain of application, which is based specially on pragmatic considerations, though further aspects are also relevant for the individualisation of the appropriate logic within this context, such as heuristic reasoning and the nature of the domain studied.

Concerning the analyticity of logic, this seems doubtful even within the boundaries of classical logic. This leads us to themes such as the independence of the axiom of choice and of the continuum hypothesis, which do not easily fit in the category of the analytic statements (nor is the formulation of Zermelo’s axiom analytic, nor even further questions linked to the controversies and problems that it has given rise to). Higher-order logic itself - logic of higher-order and set theory - commits us to axioms of an existential trait; elementary logic, furthermore, has what we could call *synthetic features*, related to its semantics, which involves topics on set theory of a non-analytic nature.

Something similar also holds for the semantics of quantum physics, which cannot be based on the standard semantic, set theoretic notions. As Malin claims:

New quantum physics has shown us models of entities with quite different behavior. Even 'sets' of photons in a looking-glass box, or of electrons in a nickel piece are much less Cantorian than the 'set' of grains of sand. In general, a highly probabilistic 'physical infinity' looks considerably more complicated and interesting than a plain infinity of 'things' (Malin 1974, p. 36).

On the other hand, realist conceptions à la Frege and Gödel, according to which logic supplies the most general features of the universe, only seem to be defensible on largely speculative grounds (Tarski, for instance, considered them as kinds of superstition or of mysticism). Nowadays, given the proliferation of heterodox logical theories, especially the existence of infinite paraconsistent logics containing a considerable part of traditional logic, the defence of an extreme realist view becomes a difficult task.
Provenance work provides a vehicle for the expression of classical logic. In a context in which a quantum logic is in use, a quantum mechanical system becomes a Boolean model and, therefore, a quantum logic. However, quantum logic is not a complete logic, and there are many instances in which it is not applicable. In such cases, a classical logic is used to express the same information in a classical manner. This is known as the quantum/classical correspondence principle.

In summary, we can think of logic as a classical logic, a special case of which is quantum logic. In this way, we can use logic to express the same information in a classical manner. However, this is not always possible, and there are many instances in which we need to use classical logic in order to express the same information in a classical manner.

These remarks raise important questions about the nature of logical reasoning and the role of logic in scientific and philosophical thinking.
Let then $x$ and $y$ be two distinct directions, and let us suppose that one has measured the momentum of $e$ in the direction $x$ and that $e_x = +1/2$; hence, $e_y = +1/2$ is true. However, as has been said, $e_x = +1/2 \lor e_y = -1/2$ is always true (in any instant). Accordingly, one can deduce that the conjunction

\[(I) \quad e_x = +1/2 \land (e_y = +1/2 \lor e_y = -1/2)\]

is also true. From (I), given the distributivity of the conjunction in relation to the disjunction, it follows:

\[(II) \quad e_x = +1/2 \land (e_y = +1/2 \lor e_y = -1/2) \iff (e_x = +1/2 \land e_y = +1/2) \lor (e_x = +1/2 \land e_y = -1/2)\]

As was seen, the left component of the biconditional is true; however, given that it is not possible to measure simultaneously the moment of $e$ in distinct directions $x$ and $y$, the right component either is false or simply meaningless. Thus, the application of classical logic leads to difficulties.

There are several possible ways to try to maintain traditional logic and to overcome the problem presented by (II). Nevertheless, thus far none of them has received unanimous acceptance. (It simply does not work if one proposes to change standard quantum mechanics for another theory, for it is of the former that we are talking about; it is also not enough to note that it is the measurement that 'creates' the spin's value, and thus the proposition '$e_x = +1/2 \lor e_y = -1/2$' is not true or false, for such a remark is against classical logic etc.)

The issue concerning the possibility of applying the category of equality to elementary particles is really delicate, and its solution does not seem to be simple. Both traditional set theory and the mathematics constructed within it presuppose the theory of equality. It follows, therefore, that a collection of electrons, for instance, does not constitute a set in the classical sense.

To sum up, there are considerable hindrances facing the possibility of applying classical logic to quantum mechanics.

Even the semantics of this theory gives rise to difficulties, given that the standard semantic methods are elaborated within traditional set theory. Such a situation is already considered even in good logic textbooks, such as Manin's:

Analyzing quantum mechanical phenomena reveals a profound divergence between the internal logical structure of the macro-world and the micro-world. Although explanations of these differences by means of natural language and natural logic are agonizingly difficult and, in the last analysis, always leave one feeling unsatisfied, these attempts to explain continue. The development of these foundations of physics in the twentieth century has taught us a serious lesson. Creating and understanding these found-

All this discussion supplies evidence for the thesis that logic, at least concerning its applications, is not bounded to entirely a priori constraints. Indeed, the criteria guiding its applications are the same that direct the applications of any mathematical theory, for instance, they are similar to those corresponding to pure geometry.

Finally, we would like to point out that a new kind of quantum logic has been proposed by Dalla Chiara and Giuntini: paraconsistent quantum logics (cf. Dalla Chiara and Giuntini 1989). These are weak forms of quantum logic, in which the non-contradiction and the excluded middle principles do not hold. As the authors argue, these logics can be seen as a 'logical abstraction' from the class of all effects in the operational approach to quantum mechanics, having also some interesting applications to this domain.

4. Paraconsistency: some technical applications

Having briefly examined some theoretical traits of paraconsistent logic, in what follows we shall consider some technical developments within the paraconsistent framework. Our main point now is to indicate some striking features that one finds when such a logic is employed as the underlying logic of mathematical reasoning. On the one hand, though this might not be that surprising, given its main traits as a particular non-classical logic in which the principle of non-contradiction is somehow restricted, some fairly unusual results (at least regarding our classical intuition) are obtained -but of a remarkable interest. On the other hand, and in straight connection to this point, despite the nature of such results, the pattern and the kind of reasoning involved in order to reach them are quite standard, indeed fully similar to current mathematical practice. Though one may study within a paraconsistent framework the properties of 'contradictory' objects, it is simply not the case that 'anything goes'. One just cannot prove anything one wishes about them. In spite of being contradictory, such objects, as it were, are not trivial. Just as in the case if the standard ones studied within the classical branches of mathematics, this kind of object has the same independence from our thoughts and desires: some properties hold of them, and some absolutely not.
THEOREMA: Theorem 1 (9691)

N. DA COSTA, O. BUNEO
PARACONSTRUCTION: A TEMPTATIVE INTERPRETATION

Proof. A theorem 1 (Russell's set x, for a universal class, A.

DEFINITION 1. (Russell's set A, for a universal class, A.

4.1. Russell's set and paraconstruction

We will briefly present a semantic analysis of Russell's set and paraconstruction.

THEOREMA: Theorem 2 (9691)

N. DA COSTA, O. BUNEO
PARACONSTRUCTION: A TEMPTATIVE INTERPRETATION

Proof. A theorem 2 (Russell's set x, for a universal class, A.

DEFINITION 2. (Russell's set A, for a universal class, A.

4.2. Russell's set and paraconstruction

We will briefly present a semantic analysis of Russell's set and paraconstruction.

THEOREMA: Theorem 3 (9691)

N. DA COSTA, O. BUNEO
PARACONSTRUCTION: A TEMPTATIVE INTERPRETATION

Proof. A theorem 3 (Russell's set x, for a universal class, A.

DEFINITION 3. (Russell's set A, for a universal class, A.

4.3. Russell's set and paraconstruction

We will briefly present a semantic analysis of Russell's set and paraconstruction.

THEOREMA: Theorem 4 (9691)

N. DA COSTA, O. BUNEO
PARACONSTRUCTION: A TEMPTATIVE INTERPRETATION

Proof. A theorem 4 (Russell's set x, for a universal class, A.

DEFINITION 4. (Russell's set A, for a universal class, A.

4.4. Russell's set and paraconstruction

We will briefly present a semantic analysis of Russell's set and paraconstruction.

THEOREMA: Theorem 5 (9691)

N. DA COSTA, O. BUNEO
PARACONSTRUCTION: A TEMPTATIVE INTERPRETATION

Proof. A theorem 5 (Russell's set x, for a universal class, A.

DEFINITION 5. (Russell's set A, for a universal class, A.

4.5. Russell's set and paraconstruction

We will briefly present a semantic analysis of Russell's set and paraconstruction.

THEOREMA: Theorem 6 (9691)

N. DA COSTA, O. BUNEO
PARACONSTRUCTION: A TEMPTATIVE INTERPRETATION

Proof. A theorem 6 (Russell's set x, for a universal class, A.

DEFINITION 6. (Russell's set A, for a universal class, A.

4.6. Russell's set and paraconstruction

We will briefly present a semantic analysis of Russell's set and paraconstruction.

THEOREMA: Theorem 7 (9691)

N. DA COSTA, O. BUNEO
PARACONSTRUCTION: A TEMPTATIVE INTERPRETATION

Proof. A theorem 7 (Russell's set x, for a universal class, A.

DEFINITION 7. (Russell's set A, for a universal class, A.

4.7. Russell's set and paraconstruction

We will briefly present a semantic analysis of Russell's set and paraconstruction.

THEOREMA: Theorem 8 (9691)

N. DA COSTA, O. BUNEO
PARACONSTRUCTION: A TEMPTATIVE INTERPRETATION

Proof. A theorem 8 (Russell's set x, for a universal class, A.

DEFINITION 8. (Russell's set A, for a universal class, A.

4.8. Russell's set and paraconstruction

We will briefly present a semantic analysis of Russell's set and paraconstruction.

THEOREMA: Theorem 9 (9691)

N. DA COSTA, O. BUNEO
PARACONSTRUCTION: A TEMPTATIVE INTERPRETATION

Proof. A theorem 9 (Russell's set x, for a universal class, A.

DEFINITION 9. (Russell's set A, for a universal class, A.

4.9. Russell's set and paraconstruction

We will briefly present a semantic analysis of Russell's set and paraconstruction.

THEOREMA: Theorem 10 (9691)

N. DA COSTA, O. BUNEO
PARACONSTRUCTION: A TEMPTATIVE INTERPRETATION

Proof. A theorem 10 (Russell's set x, for a universal class, A.

DEFINITION 10. (Russell's set A, for a universal class, A.
4.2. A paraconsistent Boolean algebra

Within various paraconsistent set theories of a certain kind, it is possible to consider intuitively a set as an ordered pair, in the classical sense, of sets that are part of a universe-set $V$. Thus, a set $X$ is the pair $<X_1, X_2>$, where:

1. $x \in X$ if, and only if, $x \in X_1$;
2. $x \notin X$ if, and only if, $x \in X_2$;
3. $x \in X$ and $x \notin X$ is equivalent to $x \in X_1$ and $x \in X_2$.

Given that the principle of the excluded middle is maintained in certain paraconsistent set theories, it should be the case that $X_1 \cup X_2 = V$. If $X_1 \cap X_2 = \emptyset$, a classical set is obtained.

Let us consider then the collection of the sets just constructed on $V$, which shall be denoted by $\mathcal{V}$. An element of $\mathcal{V}$ is called a paraconsistent set, or a p-set. In what follows we shall outline an algebra of p-sets $\mathcal{V}$. We will suppose that the p-sets are embedded in a classical set theory, for instance, ZF.

Definition 1. (Union) If $X = <X_1, X_2>$ and $Y = <Y_1, Y_2>$, then $X \cup Y = <X_1 \cup Y_1, X_2 \cap Y_2>$.

Definition 2. We shall denote by $I$ the pair $<\emptyset, \emptyset>$, and by $0, <\emptyset, V>$.

Theorem 1. All the following identities hold:

- $X \cup X = X_1$;
- $X \cup Y = Y \cup X$;
- $(X \cup Y) \cup Z = X \cup (Y \cup Z)$;
- $I \cup X = I$;
- $0 \cup X = X$.

Definition 3. (Intersection) If $X = <X_1, X_2>$ and $Y = <Y_1, Y_2>$, then $X \cap Y = <X_1 \cap Y_1, X_2 \cap Y_2>$.

Theorem 2. The following are some of the properties of the intersection:

- $X \cap X = X_1$;
- $X \cap Y = Y \cap X$;
- $(X \cap Y) \cap Z = X \cap (Y \cap Z)$;
- $I \cap X = X_1$;
- $0 \cap X = 0$.  

Theorem 5. $R \subseteq R$. 

Given theorem 5, it is possible to demonstrate that $R$ is, as it were, a 'internal model' of the set theory in which we work. Moreover, given that $\cup R = V$, it follows that the existence of $R$ implies the existence of infinite sets.

The properties of $R$ are by no means arbitrary. Thus, it is not possible to prove everything with regard to $R$, without also proving, at the same time, that some of the classical, standard set theories are inconsistent (see da Costa 1986, and also da Costa 1964).

Besides $R$, it is not difficult to introduce and to study Russell’s relations:

$x_1, x_2, ..., x_n \in R_{n,1} \leftrightarrow <x_1, x_2, ..., x_n> \notin x$.

It is easy to prove that:

Theorem 13. $R_{n,1} \subseteq R_{n,1} \cap R_{n,1} \subseteq R_{n,1}$.

Theorem 14. $V \times V \times ... \times V = \cup R_{n,1}$, where the product on the left has $n$ terms.

It is plain that $R_{1,1} = R$, when we make $<\emptyset> = x$. 

134 THEORIA - Segunda Época
Vol. 16/1, 2001, 119-145
Though the importance of this structure is possible to formulate several


Definition 7. A structure is called a paracompletion  


Theorem 4. There are some of the properties of the indiscernible:


Definition 8. An algebraic system that satisfies a subset of the properties of the indiscernible.
Let us now define a valuation associated with an interpretation. In order to do so, we shall present some notations:

Given a formula $F$, we shall denote by $F^*$ the formula obtained from $F$ in the following way: (1) one eliminates $→$ and $↔$ through the usual definitions, in terms either of $→$ and $∨$, or of $→$ and $∧$; (2) every negation is transported to the 'inner part' of the formula, so that it affects only atomic subformulas or negations of such subformulas of $F$.

If $I$ is an interpretation of $M$ in $S$, $ν_I$ or, simply, $ν_I$ is the associated valuation. It can be defined thus, where $F$ is a sentence of $M(S)$:

1. $ν_I(F) = ν_I(F^*)$;
2. $ν_I(G ∨ H) = 1 ⇒ ν_I(G) = 1$ or $ν_I(H) = 1$;
3. $ν_I(G ∧ H) = 1 ⇒ ν_I(¬G) = ν_I(¬H) = 1$;
4. $ν_I(¬P(k)) = ν_I(¬P(k'))$;
5. $ν_I(¬P(k)) = ν_I(P(k))$;
6. $ν_I(P(k)) = 1$ if $k ∈ P_I$;
7. $ν_I(P(k)) = 0$ if $k ∉ P_I$;
8. $ν_I(¬P(k)) = 1$ if $k ∈ P_I$;
9. $ν_I(¬P(k)) = 0$ if $k ∉ P_I$;
10. $ν_I(∀x G(x)) = 1 ⇒ ν_I(G(k)) = 1$, for every name or constant $k$;
11. $ν_I(∃x G(x)) = 0 ⇒ ν_I(G(k)) = 1$, for some name or constant $k$;
12. '∀x G(x) = 1' is defined in the usual way;
13. '∃x G(x) = 0' is also defined in the usual way.

As is plain, in this definition, one supposes that $k$ denotes a name or an individual constant, that $I$ assigns $P$ to $P^*$ etc.

Thus, one defines in $M$: $I, ν_I F$ if, for every interpretation $I$ and valuation $ν_I$, $ν_I(F) = 1$.

Given a formula $F^*$, one denotes by $F^{*0}$ the following formula: one replaces, in $F^*$, every occurrence of $P$ for $P_1$ (new predicate symbol) and of $¬P$ for $P_2$ (new predicate symbol, other than $P_1$) etc. Hence for every predicate symbol $P$ of $M$, we associate two new predicates, $P_1$ and $P_2$; for $Q$, we associate $Q_1$ and $Q_2$ etc.

Let us add to $M$ a new implication symbol, $⇒$, semantically characterised by the classical condition: $ν(G ⇒ H) = 1$ if, and only if, $ν(G) = 0$ or $ν(H) = 1$. Moreover, let us also add to $M$ the new predicate symbols $P_1$ and $P_2$, $Q_1$ and $Q_2$,..., admitting that in no formula are there occurrences of $⇒$ within the scope of negations. Then, an axiomatic for $M$ is the following:

1. A system of postulates for classical positive uniform calculus, relative to $⇒, ∧, ∨, ∀, ∃$.

$(2) P_1 ∨ P_2, Q_1 ∨ Q_2$...
$(3) F^{*0} / F$, for every formula in which there is no occurrence of $⇒$.

It follows that: $I, ν_I F ⇔ I, ν_I F$.

We briefly state some theorems of $M$:

1. $I, ν_I F ∧ ¬F ⇒ G$
2. $I, ν_I (F ∧ ¬F)$
3. $I, ν_I F ∨ (¬G)$
4. $I, ν_I (∀x (¬(Px ∧ ¬Px))$
5. $I, ν_I (¬Fx)$
6. $I, ν_I (∀x (Px ∨ ¬Px))$

However, the rule and the sentences below do not hold in $M$:

$F, F ⇒ G / G$
$Pa ∧ ¬Pa ⇒ G$
$Pa ∧ ¬Pa$

The predicates $R$ such that $R_1 k ∧ R_2 k$ are satisfied by no $k$ in $V$ are called classical. In this case, $ν(Ra ∧ ¬Ra) = 0$, for every $a$ in $V$, and $R$ has a classical behavior.

As just presented, $M$ consists in a starting point in order to develop, for instance, a paraconsistent syllogistic - just as the one that shall be described in the next section. Furthermore, it can also be employed as the foundations for a syllogistic whose nature was outlined by N.A. Vasil’ev, one of the forerunners of paraconsistent logic. (For an exposition of his views and references to his works, see Arruda (1984).)

5. A case study: syllogism and paraconsistency

In this section, an application of the general framework supplied by paraconsistent logic will be made to that which is perhaps one of the most ancestral domains of traditional logic: the theory of syllogism. The main point consists in addressing the issue regarding the employment of a paraconsistent logic in order to articulate such a theory, and examining which of the traditional inferences still hold.

After briefly reviewing, in section 5.1, some aspects of classical syllogistic, in section 5.2, we shall concisely present some possible answers to this issue.
the parsonsian prediction procedure ($C$). In order to reach that, it suffices that there
be a process that, in its operation, the monadic calculus corresponds to
clausal modal predicate calculus is impossible to develop a consistent modal predicate calculus, it is possible to develop a consistent system of traditional predicate logic which was integrated within

5.4. Prepositional Propositions

(3) The theory of prepositional propositions forms an integral part of the logical
structure of the sentence. Thus, the logical structure of the sentence is
the logical structure of the proposition, and not the logical structure of
the proposition. This is the basis for the development of propositional
systems of traditional predicate logic.

5.4.1. Classical Propositional Calculus

(4) The theory of classical propositional calculus provides the basis for the
development of classical logic. The classical propositional calculus is
the formalism of classical logic. The classical propositional calculus
is the formalism of classical logic. The classical propositional calculus
is the formalism of classical logic. The classical propositional calculus
is the formalism of classical logic. The classical propositional calculus
is the formalism of classical logic.

5.4.2. Logical Connectives

(5) The logical connectives are the symbols that represent the logical
operations of classical logic. The logical connectives are the symbols
that represent the logical operations of classical logic. The logical
connectives are the symbols that represent the logical operations of
classical logic. The logical connectives are the symbols that represent
the logical operations of classical logic. The logical connectives are
the symbols that represent the logical operations of classical logic.

5.4.3. Truth Tables

(6) The truth tables for classical propositional calculus are used to
evaluate the truth values of the connectives. The truth tables for
classical propositional calculus are used to evaluate the truth values
of the connectives. The truth tables for classical propositional calculus
are used to evaluate the truth values of the connectives. The truth
tables for classical propositional calculus are used to evaluate the
truth values of the connectives. The truth tables for classical
propositional calculus are used to evaluate the truth values of the
connectives.
one translate the propositions \( A, I, E \) and \( O \) into \( C^* \); the translations are formally the same as the ones just presented in the last section, which were based on the classical setting.

There are two brief remarks to be made within this context. (1) The valid positive deductions in \( C^*_o \), the classical predicate calculus, are also valid in \( C^*_1 \); that is, when no explicit negation is involved, the positive deductions of \( C^*_o \) and \( C^*_1 \) are the same. (2) In \( C^*_1 \), one can find 'paraconsistent' predicates, such that, for instance, there are elements that satisfy the predicate and, at the same time, do not satisfy it; i.e., for some predicate \( p \) the following holds:

\[ \exists x (p(x) \land \neg p(x)). \]

Thus, based on arguments rather similar to the ones found in the classical case, it is possible to verify the validity of inferences, and one changes accordingly the theories of opposition, conversion, immediate inferences and syllogism. (Each predicate within the universe of discourse has three parts: of the elements that satisfy it, of those that do not satisfy it, and of those that simultaneously satisfy it and do not satisfy it. Simple graphics supply then evidence for the validity, or for the invalidity, of certain inferences and conversions.)

Based on this approach, one can prove the following result. In the paraconsistent logic \( C^*_1 \), all modes of the first and of the third figures of the syllogism are valid; none of the second is valid; and of the fourth, just Bramantip and Dimaris modes are valid.

It is worth mentioning that \( C^*_1 \) has a strong negation, of a classical trend, and if such negation is adopted in the interpretation of syllogistic reasoning, the classical theory is obtained.

As is known, Lukasiewicz has axiomatised the theory of categorical syllogism, based on the classical propositional calculus and admitting as specific axioms certain categorical propositions, as well as some appropriate definitions. Based on the paraconsistent propositional calculus, for instance, the calculus \( C_1 \) (cf. da Costa 1974), it is also possible to formulate an axiomatics for paraconsistent syllogistic, articulated in parallel lines to the theory just outlined. Moreover, we should note that there are further extensions or modifications of the Aristotelian syllogistic that also admit paraconsistent versions, such as Hamilton's, De Morgan's and Gergone's.

6. Concluding remarks

From the previous remarks, several conclusions that outline a particular view of paraconsistent logic and, in general, of contemporary logic might be drawn.

We shall just briefly point out some of them here, leaving their development for further works.

(1) Paraconsistent logic, as opposed to the classical one, despite being a logic which allows us to examine the properties of 'contradictory objects', such as Russell's set, does not lead us to trivialisation, and moreover it is simply not the case that these objects have every imaginable feature. To some extent, they behave just as normally as other standard classical objects.

(2) The tentative points suggested here shall indicate that paraconsistent logic is philosophically neutral, in the same sense that, for instance, mathematics is. The latter, just as the former, cannot justify by itself any metaphysical or, in general, 'speculative' position. (It goes without saying, however, that logic and mathematics as well as the activity of logicians and mathematicians are subject to philosophical interpretations.)

(3) In this regard, we would like to stress however that one cannot prove that 'speculative' philosophical interpretations of paraconsistent logic cannot be true (though it might be also difficult to show that they are). Our interpretation, nevertheless, not being committed to such 'speculative' approaches, seems to be philosophically more acceptable.

(4) Once the distinction between pure and applied logic is made, it seems natural to claim that the latter is not restricted exclusively to \textit{a priori} considerations, but depends on the particular features of the domain to which it is applied (or on the propositional way of representing the latter). As von Neumann claimed:

\begin{quote}
The basic idea is that the system of logics which one uses should be derived from aggregate experiences relative to the main application which one wishes to make -logics should be inspired by experience. (von Neumann [1937], p. 2)
\end{quote}

Acknowledgements

We wish to thank Steven French for comments on an earlier version of this paper. Thanks are also due to two anonymous referees for their helpful suggestions.