REVIEW

ANDRÉ FUHRMANN

An Essay on Contraction

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The representation of theory change in science spans a range of approaches, with, at one extreme, the formal work of Alchourrón, Gärdenfors and Makinson (AGM). The basic problem is as follows: representing theories sententially, how are they to be adjusted when faced with some new item of information? There are three major possibilities: expansions, which result in a larger theory, contractions, which result in a smaller one, and revisions, which result in a theory which is neither an extension, nor a part, of the original theory. The AGM approach tackles these in terms of sentence-by-sentence adjustments. Expansions are easiest to deal with: if the item of information is consistent with the theory, simply add it and close under logical consequence. Contractions and revisions are trickier and more interesting and, under the umbrella of a maxim of minimum mutilation, the famous AGM postulates were laid down to deal with them. An important result by Gärdenfors established early on the interdefinability of contractions and revisions. So it is largely a matter of choice which notion to take as primitive. In the literature, a great deal of attention has been given to contraction, and in particular to two ways of constructing contraction functions: through a relation of epistemic entrenchment, which involves a ranking of the sentences of the theory, or via 'partial meet' contractions involving a selection function on the family of sub-theories of the theory in question.

Fuhrmann follows the tradition of focusing on contractions. But he offers a wide ranging generalisation of the AGM theory, showing how to obtain contractions of arbitrary sets of sentences by arbitrary sets of sentences. The
overall framework is provided by James’s description of belief change which emphasises that old opinions are ‘married’ to new facts ‘so as ever to show a minimum of jolt, a maximum of continuity’ (quoted on p. 1). Within such an overarching view, Fuhrmann stays with the pragmatists in adopting Peirce’s ‘belief-doubt’ model of inquiry which holds to a form of cognitive conservatism in retaining beliefs until forced to retract them and then to adjust only through minimal incisions into the belief set.

Following a clear exposition of the AGM theory, the heart of the book lies in Chapter 3, where Fuhrmann considers the contraction of sets of sentences which are not closed under consequence and contraction by sets of sentences which are not singletons. The former is dealt with by the notion of ‘base contraction’, which captures the plausible idea that what is contracted is never the theory as a whole, but rather some distinguished subset, or base, of the theory. It then follows that if some component of the base is removed, then so should any sentences that were dependent on that component.

The second case, that of ‘multiple contractions’ covers examples such as the simultaneous alteration of many parts of a legal code, or a scientific theory, and interestingly, the inadvertent expansion of a belief set into inconsistency where the best move is to retract both conjuncts of the contradiction. The latter also provides an example of what Fuhrmann calls ‘package’ contraction, where all members of a set are retracted; there is also the possibility of ‘choice’ contraction where only certain elements of the set are retracted. Fuhrmann believes that not only are multiple contractions ubiquitous, but they are also necessary and suggests, although he declines to follow this up here, that they are rational. He also explores the possibility that such multiple contractions can be reduced to singleton ones and concludes that although for finite sets reduction goes through for choice contractions, it fails for package; moreover, contraction by an infinite set cannot in general be simulated by retracting a finite set (p. 58).

As in the previous discussion, multiple contractions can be accommodated indirectly by suitable generalizations of the AGM postulates or directly by giving ‘recipes’ for the construction of both package and choice contractions. The generalization of the AGM postulates is straightforward with the postulate of ‘recovery’—which expresses the above maxim of minimum mutilation and demands that if a removed sentence is reinserted into the contracted belief set, then the original set is recovered, a demand that is contentious without the requirement of closure—replaced by that of ‘relevance’, which demands that ‘whatever is being removed from a belief set in the course of a contraction does in some way contribute to entailing the sentence to be retracted’ (p. 43). Moreover, the remainder operation is also generalized to package and choice remainders. (The maximal subsets of a theory T that do not entail a sentence a are the remainders of T after removing a.) The generalized operation acts on two arbitrary sets of sentences rather than one closed set and a singleton set. As a result, a broader set of postulates is provided, and Fuhrmann shows that given a theory as ‘input’ the postulates still produce a theory as ‘output’. Package and choice contractions are also constructed directly within the constraints provided by the postulates by means of appropriate generalizations.

The chapter concludes with an interesting discussion of the problem posed by Wittgenstein: ‘what is left over if I substract the fact that my arm goes up from the fact that I raise my arm?’ (quoted on p. 65). Tackling this issue in terms of a kind of inverse of conjunction (the subtraction connective) runs into the problem of contradictory definitions for negative definitions. Instead, Fuhrmann notes that Jaeger’s constraints on a satisfactory subtraction connective are analogous to his own contraction postulates, suggesting that subtraction can be understood as sentential contraction. A possible worlds semantics for subtraction is then presented by adapting Grove’s possible worlds models of theory change for contractions.

The aim of Chapter 4 is twofold: first to develop a theory of general revisions and merges; secondly, to establish a connection between belief change and inference. The difference between revisions and merges is that whereas with the former, the sentences to be revised by take precedence over those to be revised, in the latter there is no such asymmetry. Fuhrmann shows that the theories of revisions is interderivable with that of contractions and then that the merge operation can be defined either in terms of contractions or of revisions. Merge is possibly the more interesting operation since, as Fuhrmann notes, it reflects more directly the Jamesian account of belief change and allows for the possibility that one’s old beliefs may be sufficiently well entrenched as to take precedence over new ones. The central idea is to understand the merge of two sets in terms of the deductively maximally consistent subset of their union, where, using the tools deployed in the previous chapter, a selection function is defined to pick-out those deductively maximally consistent subsets which are more valuable to the enquirer (p. 81).

It might be interesting at this point to consider weakening the consistency constraint to allow for the merge operation to yield inconsistent belief sets, even if it is then acknowledged that these are simply stages on the road towards a consistent successor. (In this case, we would need to change the underlying logic into a paraconsistent one.) Take that old chestnut, Bohr’s 1913 theory of the atom. One could view this, à la Fuhrmann, as the result of a merge of classical mechanics and (old) quantum postulates (corresponding to Fuhrmann’s ‘worst case’ (ibid.) of a maximally inconsistent union) yielding an inconsistent subset. Granted the point just acknowledged, such actual episodes of belief change are regarded by some of us as rather more common than others suspect! Fuhrmann, however, wants to accommodate inconsistency without having to resort to non-classical logic.
This is apparent in the rest of the chapter, where he incorporates defeasible inference into his framework. He begins by noting that default reasoning faces the problem of ‘multiple extensions’: there is usually more than one way of extending the explicit premises by as many defaults as are consistent with these premises (p. 86). Standard approaches to this problem, such as that of Gärdenfors and Makinson, incorporate the asymmetry noted above: whereas the set of new facts or premises is kept unchanged, the set of defaults is manipulated so as to be consistent with the latter. As Fuhrmann notes, this not only violates the Jamesian approach, it also fails to deal with the possibility that the set of new facts may be inconsistent in the first place. This is an important issue in AI, for example, where one might be attempting to formalize procedures for dealing with very large sets of ‘facts’, such that simple consistency checks are just not feasible. One approach, as Fuhrmann notes, is to adopt a non-classical, paraconsistent logic, as suggested by da Costa. Fuhrmann himself prefers a more classical approach based on his merge operation which removes the epistemic imbalance between background hypotheses and explicit premises and thus supplies a tool for drawing non-trivial conclusions from inconsistent sets of premises (p. 91). The upshot is a relation of ‘merge inference’ denoted by $\vdash$ which, interestingly, sacrifices reflexivity ($a \vdash a$) in favour of ‘right weakening’

$$ A \vdash a_1, a_1 \vdash a_2. $$

$$ A \vdash a_2. $$

As Fuhrmann notes, this satisfies almost all of da Costa’s desiderata for paraconsistent logic, with the exception of the condition that the principle of contradiction must not be a valid schema. Indeed, he continues, it satisfies da Costa’s final condition even better than most paraconsistent logics; this is a condition that distinguishes da Costa systems from, say, Priest’s and states that such logics must contain as much of classical logic which does not conflict with the other conditions. Merge inference behaves classically with consistent premises and where these are inconsistent effectively implements the maxim of minimal mutilation by salvaging the maximum amount of reliable information (ibid.).

However, as Fuhrmann then notes, ‘Merge inference presupposes that maximally consistent subtheories of a given inconsistent theory can be determined and weighted. If either one of these presuppositions cannot be met, then only closure operations, as provided by paraconsistent logics, can avert the threat of triviality’ (p. 92). Consider again Bohr’s theory. Contrary to the claims of the advocates of the non-adjunctive approach, it is actually contentious whether maximally consistent subtheories of Bohr’s theory can even be determined, much less weighted. One problem is whether in reducing a theory to consistent sub-blocks one still has the same theory. Given this, simply acknowledging the inconsistency and moving to a paraconsistent logic may be the more appropriate option. Another alternative, which Fuhrmann might be more sympathetic to, would be to effectively drop the distinction between ‘theories’, which are logically closed, and ‘open’ belief sets and treat Bohr’s theory as interestingly open. As Fuhrmann notes, AI research tends to be conservative in going for the classical approach and ‘pairing down inconsistent blocks of information into consistent sub-blocks’ (p. 94), before adjudicating between them. This might be appropriate when one can clearly identify the ‘blocks’ to begin with, but not all cases fit this schema so straightforwardly.

The final chapter, entitled ‘Everything in Flux: Dynamic Ontologies’, goes even further in mapping out some possible future lines of research. Fuhrmann begins by noting that one can define a relation of ‘requirement’ for an ontology $O$ (viewed as a possible collection of objects) in the sense that the presence of objects in $O$ may not be possible without the presence of another object $a$. In other words, for $O$ to constitute an ontology certain members of that collection are required. Transitions from one ontology to another can then be represented by expansion and contractions in ‘ontological space’ constituted by a domain of objects together with an appropriate requirement relation. The former can be represented by the closed union of the collection and object concerned, while ontological contractions can be handled in terms of the framework of earlier chapters, generalised to arbitrary closure systems. ‘Good’ examples in Fuhrmann’s terms (that is those that are interesting or non-trivial) are provided by actions, where choice may be required for some further action to be performable; things, where contraction from the fusion of two individuals may require choice; situations of comparative necessity, where some objects can be more easily deleted from an ontology than others, and functional dependency, where a given attribute may no longer have a determinate value for some object, requiring, in turn, a determination of what other attributes can no longer be assumed determinate. The latter issue is of particular importance in the design of databases and Fuhrmann suggests his general framework for contraction can handle all these examples.

Although this work may seem to have only a restricted interest, we hope we have indicated that its scope covers in fact a considerable range. On the whole, it is accessible and well written (although there are a few editorial errors) and although formal approaches to belief change may not be everyone’s cup of tea, this offers a heady brew of philosophically informed rigour and originality.