Chapter 1
Overcoming Newman’s Objection

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Abstract Bertrand Russell (The analysis of matter. Routledge, London, 1927) defended a form of structuralism according to which all we can know about the world is structure. In response, Max Newman (Mind 37:137–148, 1928) raised a formidable challenge that threatens to turn structuralism into something trivial: as long as there are enough objects in the relevant domain, one can always obtain a structure suitable for that domain. In this paper, I consider two responses to this objection. The first is provided by Rudolf Carnap (The logical structure of the world. Trans. Rolf A. George. University of California Press, Berkeley, 1928/1967) in terms of founded relations. I argue that it ultimately fails. Another alternative insists that the structures that have content about the world are ultimately finite, and it is a non-trivial matter to figure out what the appropriate structure for finite domains ultimately is. Russell (The autobiography of Bertrand Russell, vol 2. Allen & Unwin, London, 1968, 176) briefly considered this option in his response to Newman, but did not develop it further. I argue that, when coupled with a proper account of detectable relations, it is a far more promising route than it may initially seem.

Keywords Structuralism • Newman’s objection • Carnap • Russell

1.1 Introduction

In The Analysis of Matter, Bertrand Russell defended a form of structuralism according to which all we can know about the world is structure (see Russell 1927). At that point, Russell has been flirting with structuralism for quite some time. Since at least The Problems of Philosophy, originally published in 1912, he also advocated structuralist views (Russell 1912). In response to this form of structuralism, Max Newman (1928) raised a formidable challenge. The objection threatens to turn structuralism into something trivial: as long as there are enough objects in the
relevant domain, one can always obtain a structure suitable for that domain. As Newman points out:

Any collection of things can be organized so as to have the structure $M$, provided there are the right number of them. Hence the doctrine that only structure is known involves the doctrine that nothing can be known that is not logically deductible from the mere fact of existence, except (‘theoretically’) the number of constituting objects (Newman 1928, 144).

But our knowledge of the world—even structural knowledge of it—is supposedly nontrivial. (For additional discussion Newman’s objection, see Ladyman 1998; Melia and Saatsi 2006; Ainsworth 2009.)

I consider two responses to this objection. I criticize the first, and defend the second. One is provided by Rudolf Carnap’s version of the objection that he considers in the *Aufbau* (Carnap 1928/1967, section 154; see Demopoulos and Friedman 1985 for an early discussion). Carnap suggests that the triviality can be avoided by requiring that the relations in question be *founded*. Surprisingly, however, he considers a founded relation as a basic concept of *logic*. I argue that, interpreted in this way, Carnap’s solution fails.

Another approach is to note that the relevant structures that have content about the world are ultimately *finite* (even though they may be, and typically are, embedded in infinite structures), and it is not a trivial matter to determine what the appropriate structure for finite domains ultimately is. This is a suggestion that Russell briefly considers in his own response to Newman (Russell 1968, 176), but, unwisely, does not develop it further. It is a far more promising route than it may initially seem.

I also argue that the Newman objection fails whether we consider first-order or higher-order logic. As it turns out, we can fix the cardinality of finite domains in first-order logic (or higher-order logic for that matter) as long as we are dealing with finite structures. If infinite structures are involved, even the notion of finiteness becomes indeterminate, given the existence of nonstandard models (Bueno 2005). The problem emerges even for higher-order logics, given Henkin semantics for these logics, which ultimately allow for nonstandard models for structures with infinite domains (Shapiro 1991). The concern, however, does not emerge for finite structures (of a particular, detectable sort, which will become clear below). In contrast with Newman’s concern, it is a substantial, nontrivial, empirical matter to determine which structure a finite physical domain has.

### 1.2 Founded Relations

In the *Aufbau*, it is clear that Carnap is concerned with a potential trivialization of the overall project. In particular, he examines a version of the Newman objection in the context of his constructional system (Carnap 1928/1967, section 154). The issue emerges from the possibility of replacing the basic, non-logical relations of the system with different basic relations, and the concern is that the resulting
system may no longer have empirical significance since the new basic relations may be entirely arbitrary and not bear connections to the empirical world. To address this potential difficulty, Carnap assumes that the new basic relations are “natural”, “experienceable” relations, and thus non-arbitrary (since they are natural) and empirically significant (for they can be experienced). As he notes:

The task of eliminating the basic relations as the only nonlogical objects of the constructional system contains one more difficulty to which we have to pay some further attention. We had assumed that, after a replacement of one set of basic relations by another, the constructional formulas of the system would not remain applicable, and the empirical statements would cease to hold. However, our assumption is justified only if the new relation extensions are not arbitrary, unconnected pair lists, but if we require of them that they correspond to some experienceable, “natural” relations (to give a preliminary, vague expression) (Carnap 1928/1967, 235–236).

So, on Carnap’s view, as long as experienceable, natural basic relations are employed at the basis of the constructional system, the resulting system would still be empirically significant.

What would happen, however, if the experienceable-basic-relation requirement were not insisted on? Carnap considers this possibility. He notes that, in this case, it would be possible to construct an isomorphic constructional system, but whose components would have different extensions. As a result, the relevant terms would ultimately have different referents, and being about different things, they would ultimately mean something different. As he argues:

If no such [experienceable] requirement is made, then there are certainly other relation extensions for which all constructional formulas can be produced. However, in such a case, the construction leads to other entities than with the original relation extensions, but, for these other entities, the same empirical statements still hold as for the original ones (that is to say, the symbols for these statements are still the same, but they now mean something different) (Carnap 1928/1967, 236).

After the transformation, the empirical statements made in the original constructional system are preserved, in the sense that they are still true, albeit true of different things. Quintessentially philosophical (!) statements such as “The cat is on the mat” and “The cherry is on the tree” have the same form, and both come out true if cats and cherries, mats and trees are mapped into one another while the relation between each of these components is preserved.

In the case of the constructional system, the statements are only concerned with formal properties, and by suitably mapping the relevant basic elements into one another the resulting transformation can be implemented. As Carnap points out:

All we have to do is to carry out a one-to-one transformation of the set of basic elements into itself and determine as the new basic relations those relation extensions whose inventory is the transformed inventory of the original basic relations. In this case, the new relation extensions have the same structure as the original ones (they are “isomorphic”, cf. section 11). From this it follows that, to each originally constructed object, this corresponds precisely one new one with the same formal properties. Thus all statements of the constructional system continue to hold, since they concern only formal properties (Carnap 1928/1967, 236).
It is via this mapping of the basic elements of the constructional system that an isomorphic system is obtained. Given the isomorphism between the two systems, they are elementarily equivalent, and thus the same statements are true in them. It is this structure-preserving argument that is at the core of the version of Newman’s objection in the Aufbau. And since no constraints have been imposed on basic elements, the resulting system lost any empirical significance.

However, we can then not find any sense for the new basic relations; they are lists of pairs of basic elements without any (experienceable) connection. It is even more difficult to find for the constructed objects any entities which are not in some way disjointed (Carnap 1928/1967, 236).

At this point, an important difference between Newman’s objection and Carnap’s transformation argument should be noted. The former is ultimately concerned with the triviality that a pure form of structuralism, which insists that our knowledge of the world is restricted to knowledge of structure, is involved with. The latter highlights the easiness of generating isomorphic systems with no contact with the world. But the reason why Carnap’s transformation argument is fruitfully conceptualized as a form of Newman’s objection emerges from the fact that it is also the result of a triviality charge: it is all too easy to obtain isomorphic constructional systems independently of their empirical content.

Carnap suggests that the triviality can be avoided by requiring that the relations in question be founded:

We wish to call relation extensions which correspond to experienceable, “natural” relations founded relation extensions. Thus, the various member pairs of founded relation extensions have something in common that can be experienced (Carnap 1928/1967, 236).

Founded relations are the key component in Carnap’s response to the triviality charge. After all, in virtue of being based on experience and being natural relations, they are ultimately grounded in the world.

Somewhat surprisingly, however, given what experienceable relations are, Carnap considers founded relations a basic concept of logic. On his view:

It is perhaps permissible, because of [its] generality, to envisage the concept of foundedness as a concept of logic and to introduce it, since it is undefinable, as a basic concept of logic. That this concept is concerned with the application to object domains is not a valid objection to introducing it as a basic concept of logic. The same is true for another basic concept of logic, namely, generality: ‘(x) f(x)’ means that the propositional function of f(x) has the value true for every argument of an object domain in which it is meaningful. Logic is not really a domain at all, but contains those statements which (as tautologies) hold for the objects of any domain whatever. From this it follows that it must concern itself precisely with those concepts which are applicable to any domain whatever. And foundedness, after all, belongs to these concepts. In view of these reasons, let us introduce the class of founded relation extensions as a basic concept of logic [...] without therefore considering the problem as already solved (Carnap 1928/1967, 237).

It is not difficult to see why, for Carnap, it is so tempting to consider founded relations as logical. He takes logical relations to be epistemologically unproblematic: they are perfectly general, content-free, and analytic. Consider, for instance, universal quantification, the example that Carnap himself mentions above. The
universal quantifier ranges over all of the objects in the domain of quantification (all of which are in the scope of the quantifier). This involves three crucial features: (a) full generality, given, of course, the domain under consideration; (b) topic-neutrality, since independently of the objects that are in the domain, universal quantification behaves in the same way, and (c) analyticity, given that the truth of statements involving logical notions alone, including, of course, quantification, are independent of the particular objects involved, and rely only on the meaning of the relevant terms.

As it turns out, empirical notions share none of these features. These notions have content, in the sense that they have a limited domain of application: they apply to certain objects and fail to apply to others. They lack the generality of logical notions, given that they are about particular objects, which typically are causally active and spatiotemporally located. As a result, empirical notions are topic-dependent: they hinge on relevant features of the objects they apply to, in contrast to logical notions’ topic-neutrality. Finally, as opposed to logical notions, empirical notions are not analytic: the truth of statements in which they figure does not depend only on the meaning of the relevant expressions; the contribution of particular objects in the world is crucial. Given these features, when successfully applied to appropriate objects in the empirical world, empirical notions can be used to rule out particular contingent configurations in reality. After all, they can be used to show that, given the way the world is, a certain configuration obtains, and thus those incompatible with it do not. In many instances, particularly when one deals with scientific results, it is a substantive achievement to specify which of these configurations, among all that could obtain, actually do obtain.

Logical notions are unable to do that, since they are compatible with all logically possible configurations, at least on the traditional conception of logic (a conception that Carnap seems to endorse). This is part and parcel of the fact that logical notions are topic neutral and content-free, given that they are analytic and fully general. And even if these notions had some content, for instance by being non-analytic, and were in some way dependent on the topic at hand, their generality would ensure that their ability to rule out possibilities beyond what is inconsistent with logical principles is null. Based on logical notions and their corresponding principles, one could rule out only logically inconsistent configurations (assuming classical logic), but nothing beyond that.

Understood in this way, Carnap’s own attempt to block a version of Newman’s triviality objection blatantly fails. As Carnap acknowledges, a logical relation provides no constraint at all on the structures under consideration, precisely because of its generality. “Logic”, he tells us, “is not really a domain at all” (1928/1967, 237). To offer an answer to Newman’s objection, what is needed is something with content. But it is unclear that a logical notion, as understood by Carnap, can provide this. A founded relation needs to be experienced rather than taken to be a logical notion for it to be able to do the work that Carnap intends it to do.

But how can an experienced relation be a logical relation? As we saw, Carnap requires that the relevant founded relations be experienceable, natural relations (Carnap 1928/1967, 236). As such these relations have a modal force: they are the
kinds of things that, at least in principle, can be experienced. Moreover, they are naturally occurring relations; they are not just artifacts, but are part of the empirical configuration of the world. Presumably experienced relations have the three features that empirical notions have: they are particular (rather than fully general), topic dependent (rather than topic neutral), and non-analytic (rather than analytic).

Are these three features preserved when the explicitly modal notion of an experienceable relation is invoked? There is a sense in which logical relations do have an empirical bearing, and as such are experienceable. Carnap insists that logical relations are applicable to any domain whatsoever (although this means, as just noted, that logic is not a particular domain; Carnap 1928/1967, 237). Presumably, that a logical law holds in any arbitrary domain entails that no object could undermine it, which, in turn, guarantees that every configuration grounds the satisfaction of a logical law. For instance, it is the case that, for any object $a$, either $Fa$ or not-$Fa$. So in experiencing, say, that $Fa$, we thereby can also experience that $Fa$ or not-$Fa$, since nothing more is required for the satisfaction of the latter once the former is satisfied.

But this way of grounding logical laws on empirical domains deprives these laws from any capacity to rule out anything other than what is logically inconsistent (that is, incompatible with some logical law or another). So the fact that some logical relation is experienceable does not preclude that relation from obtaining no matter what. The only restrictions provided are those given by the relevant logical laws, which, following Carnap, are those of classical logic. In other words, an experienceable relation that is also a logical relation does not provide any significant restriction beyond logical consistency. As far as the empirical world is concerned, we are left with no constraints whatsoever—despite the fact that the application of some logical notions (such as the universal quantifier) may depend on empirical traits. In order to determine whether everything is $F$, one needs to check which things are $F$. But in this case, the constraints are provided by the $Fs$, whatever they turn out to be in the world, not by the quantifier.

In addition to experienceable relations, Carnap also requires that founded relations be natural (1928/1967, 236). I take this to mean that a natural relation is not just an artifact, but is ultimately grounded in the world. Now, if such a natural founded relation is also a logical relation, as Carnap seems to require (1928/1967, 237), once again it is unable to rule out any empirical configuration other than those that are logically inconsistent given classical logic. As a result, given Carnap’s way of framing the issue, no constraints are ultimately provided, and the triviality objection is not blocked.

### 1.3 Finiteness

There is a second approach to Newman’s objection that is worth exploring. The central point is to highlight that the relevant structures that have content about the world are ultimately finite. (Below I will address the point that finite structures are
often embedded in infinite ones, indicating why the focus on finiteness is crucial.) Given a finite domain, it is no trivial issue to figure out what its appropriate structure ultimately is. This is a path worth exploring in more detail.

The Newman objection fails when appropriate finite structures are considered (whether in first-order or higher-order logic). A finite structure is appropriate as long as the properties and relations in the structure correspond to detectable properties and relations in the world. What are detectable properties and relations (henceforth, detectable relations for short)? These are relations to which the empirical access is counterfactually dependent on particular features of the world. This is the case as long as two counterfactual conditions are met:

(a) Had the sample (or the scene before one’s eyes) been different, within the sensitivity range of the relevant instrument (including one’s eyes), the resulting image (or visual experience) would have been correspondingly different.

(b) Had the sample (or the scene before one’s eyes) been the same, within the sensitivity range of the relevant instrument (including one’s eyes), the resulting image (or visual experience) would have been correspondingly the same.

These two conditions establish a form of access to the relevant relations that is robust (it is stable over changes of beliefs about what is being detected), can be refined (one can get closer for a better “look”), and allows the relations to be tracked (in space and time). This form of access integrates both observations with the naked eye and instrumentally mediated observations (as long as the instruments in question provide suitable images), highlighting that these forms of observation satisfy the same dependence conditions. The result is a form of detection of empirical relations that depends on, is constrained by, and tracks relevant features of the world. (For additional discussion, see Bueno 2011; this account develops further and integrates proposals advanced in Lewis 1980 and Azzouni 2004. It would take me too far afield to address here the issue of how to detect “properties” such as grue.)

Interestingly, detected in this way are finite relations, given that one cannot visually experience infinite configurations. But this is as it should be. For what is needed is precisely a form of detection that is significantly constrained by the empirical set up, and finiteness is a crucial aspect that makes this process possible. After all, as will become clear, it is the presence of infinite structures that allows for the multiplicity of possible structures that eventually leads to Newman’s trivialization result.

Note that the cardinality of finite domains, whether in first- or higher-order logics, can be fixed if we are dealing with appropriate finite structures. These are the detectable finite structures we have empirical access to. Their finiteness allows one to track their features in a robust and refined way, determining the configuration of the relevant relations in the structures they are part of. Had the structures been infinite, such detectability would not be possible, given the limited range of what can be observed, even if instrumentally mediated observations are incorporated.

Furthermore, in the case of infinite structures, the notion of finiteness is ultimately indeterminate in light of nonstandard models: models in which “finite” is satisfied by infinite objects. If that happens, one loses the grip on finiteness,
which no longer is able to constrain the empirical set up, since infinite objects can satisfy “finite” (for further discussion, see Bueno 2005). The same difficult, mutatis mutandis, also applies to higher-order logics. After all, these logics have a Henkin semantics, and nonstandard models for structures with infinite domains emerge (Shapiro 1991). The result, once again, is the indeterminacy of finiteness in the presence of infinite structures.

This difficulty, however, is bypassed for appropriate finite structures, given that for these structures, due to the unavailability of nonstandard models, no such indeterminacy emerges. As opposed to what Newman’s objection states, it is a substantial, nontrivial, empirical matter to find out which structure a finite physical domain in fact has.

Of course, one often embeds finite structures into infinite ones. In many instances the full scope of certain scientific theories seems to require infinite structures. Space-time theories are an obvious example. In these cases, it is crucial to distinguish the mathematical framework that is used to represent the relevant physical systems, which often does incorporate and require infinite structures, and the empirical set up, which makes no corresponding requirement regarding infinity. That the underlying mathematical framework calls for infinite structures should not be a reason for one to assume that the empirical set up itself also makes a similar demand. One would be reading more into the empirical set up than is in fact warranted (an unfortunate mismatch that sometimes happens; see Bueno and French 2012 for further discussion).

The inferential conception of modeling (Bueno and Colyvan 2011; Bueno 2014) highlights how this can be avoided. We start from a finite empirical set up, and then embed the relevant finite structure into an infinite one. We then draw the relevant conclusions that are made possible by the richer structures we are operating with. The obtained results, however, do not state anything about the empirical world (they are mathematical results at this point), and they need to be suitably interpreted back into the finite empirical set up. It is crucial, at this point, to bear in mind that this set up itself has no infinite structure in it: it is a finite domain, after all. And as long as one is careful not to import unwarranted assumptions while interpreting results from infinite domains, one can avoid the difficulties of over-interpretation mentioned above. In this way, results are made relevant to the finite domain they are applied to. (For further discussion of the inferential conception of modeling, see Bueno and Colyvan 2011; Bueno 2014.)

It may be objected that as long as there are enough objects in the domain, one can always arrange them in such a way that they will exhibit the structure of the empirical set up. But in order to do that one needs first to establish what that structure is. It is not enough simply to state that it is always possible to obtain such a structure (as long as there are enough objects in the domain). After all, what needs to be settled in the first place is the particular configuration of the relevant structure, not that such a structure can always be obtained given that any possible structure can always be (provided enough objects are available).

For instance, suppose one is trying to determine what is the appropriate structure of a given molecule. It is beside the point to state that, whatever that structure
turns out to be, as long as there are enough objects in the domain, it can always be constructed. What is at issue is precisely the particular configuration of atoms that form the relevant molecule. At issue is the need to identify that structure and what it is. That there can be a structure of that sort (whatever it turns out to be) is of no help, since the identification of the structure is what is at stake, not that such a structure is possible.

Of course, if there are concerns about the possibility of (the existence of) a structure of a given sort, then the more general question, regarding the possibility of a structure, does become relevant. Perhaps certain configurations of atoms are unstable, precarious, or intractable. In this case, to establish the possibility that a given kind of structure exists does become relevant. But typically this is done by taking into account particular features of the domain under consideration, relying on relevant bits of information about the objects that are being studied; in this case, atoms and their allowable configurations. The required arguments do not result simply from some model-theoretic construction that guarantees the availability of certain structures. Arguments of this sort tend to be too general and abstract for the particular issue at hand. Not surprisingly, considerations that invoke a model-theoretic construction are fundamentally different from those raised by the initial problem, namely, what the structure of a given molecule actually is. Model-theoretic considerations operate at a different level of abstraction than those that emerge from the need to determine the actual structure exhibited by the molecule under investigation. Given the focus on finite, detectable relations (which are those needed to figure out the structure of the relevant molecule), the arbitrariness of Newman’s objection simply does not emerge.

Underlying the approach to Newman’s problem recommended here is a form of empiricism that emphasizes the significance of structures that can be detected instrumentally—via instruments that we know, or have good reason to believe, that satisfy the counterfactual dependence conditions above. As a form of empiricism, the view is agnostic about those structures that go beyond the reach of what can be so detected. Realists may insist that Newman’s problem also concerns structures that cannot be detected in the way empiricists recommend. That may be so. But this provides an additional reason to relinquish realism and adopt a robust form of empiricism instead—one that overcomes Newman’s objection where it matters.

1.4 Conclusion

By emphasizing the importance of finite, detectable relations, we can provide a robust response to Newman’s objection, one that does not face the difficulties that undermined Carnap’s approach to the issue. Clearly detectable relations are not logical relations, and they are not meant to be. As such, they have content: they are particular (they are restricted to a particular domain); they are topic dependent (their features depend on what is going on in the domains they are part of), and they are not “analytic” (that is, the truth of statements involving such relations depend
on what goes on in the empirical set up rather than just on the meaning of the terms involved). Carnap’s focus on experienceable, founded relations was on the right track, but clearly any such notion cannot just be logical; otherwise, no effective response to Newman’s objection is forthcoming.

Newman’s objection only becomes a problem if the kind of structures under consideration is left unconstrained. By focusing on appropriate, detectable, finite structures, a suitable constraint is found. Carnap sensed the need to provide some restrictions on the relevant structures to avoid triviality, and his founded relations are certainly a step in the correct direction. But all benefits are lost if founded relations are conceived of as logical relations. What is needed, instead, are detectable, empirical relations in a finite structure. And that gets the job done.

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References