Modalism and Logical Pluralism
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Logical pluralism is the view according to which there is more than one relation of logical consequence, even within a given language. A recent articulation of this view has been developed in terms of quantification over different cases: classical logic emerges from consistent and complete cases; constructive logic from consistent and incomplete cases, and paraconsistent logic from inconsistent and complete cases. We argue that this formulation causes pluralism to collapse into either logical nihilism or logical universalism. In its place, we propose a modalist account of logical pluralism that is independently well motivated and that avoids these collapses.

1. Introduction
According to logical pluralism, there is more than one genuine deductive consequence relation, even within a given language. The position is opposed to the more common view, logical monism, which privileges a single deductive consequence relation. But why should anyone be a logical pluralist? In a recent work, JC Beall and Greg Restall provide a framework to answer that question, and along the way, articulate logical pluralism in an interesting manner (Beall and Restall 2006). Central to their proposal is the preservation of a core notion of validity: an argument is valid if, and only if, in every case in which the premisses are true, so is the conclusion. They argue that by considering different types of cases the logical pluralist obtains different logics. For example, if the cases are consistent and complete (say, Tarskian models), the logics are classical. If the cases are consistent and incomplete (say, certain mathematical constructions), constructive logics emerge. If the cases are inconsistent and complete (say, certain databases), the logics are paraconsistent. In order to have a logic, however, Beall and Restall insist that the notion of logical consequence meet three familiar constraints: necessity, formality, and normativity.

In this paper, we argue that, despite its intuitive appeal, the formulation of logical pluralism in terms of cases is ultimately inadequate. There are subtle tensions within the constraints imposed on the notion of logical consequence. The logical pluralist as characterized is not in a
position to make sense of some of the constraints, particularly the necessity constraint. After defending this point, we offer an alternative understanding of logical pluralism that couples modalism—the view that some modality is treated as primitive—and attention to the demands of different domains. We argue that this understanding of pluralism avoids the difficulties raised for pluralism via cases. An independently well-motivated form of logical pluralism emerges.

2. Logical pluralism by cases: some troubles

On Beall and Restall’s view, a logic is not simply any old consequence relation in a given language. Otherwise, logical pluralism, although perhaps obviously true, would not be a very interesting position. A consequence relation must satisfy three constraints each of which derives from the tradition of logical theorizing, and has been endorsed over the centuries, in one way or another, by practitioners.

First, there is a necessity constraint. In a valid argument, the truth of the premisses necessitates the truth of the conclusion; some things are not possible—they are ruled out—given the premisses. Beall and Restall (2006, pp. 14–16) give a standard amplification of this constraint in terms of possible worlds: a claim is necessarily true if, and only if, it is true in all possible worlds. This constraint is supposed to ensure that, in valid arguments, the transition from true premisses to a true conclusion is not a matter of accident.

Second, there is the normativity constraint. When we commit a logical violation, we need to specify the nature of the mistake that is committed. Norms are, in fact, often violated and a full accounting of a logic should specify the type of mistake that is made when the norms regarding what follows from what are broken.

Third, there is a formality constraint. Following MacFarlane (2000), Beall and Restall distinguish three senses of formality: (a) Logic provides constitutive norms for reasoning, regardless of subject matter. (b) Logic does not depend on the particular identities of objects. (c) Logic abstracts from the semantic content of the statements it deals with, focusing on the non-semantic features of the statements in question. To meet the formality requirement, it is enough that a logic be formal in at least one of these senses.

For the purposes of this paper, we focus on the necessity constraint. This is a central requirement for a consequence relation and, as we will argue shortly, it raises significant difficulties for the form of logical pluralism that Beall and Restall favour. How does classical logic meet such
a constraint, on their picture? In order to answer that question, Beall and Restall consider the distinction between two accounts of validity that are often associated with classical logic (2006, pp. 36–41). On the one hand, we have the *necessary truth-preservation* (NTP) account of validity, according to which an argument is valid if, and only if, in any world in which the premisses are true, so is the conclusion. On the other hand, we have the *Tarskian model* (TM) account, according to which an argument is valid if, and only if, in every model in which the premisses are true, so is the conclusion.

According to Beall and Restall, the NTP account can be used to show that classical logical consequence meets the necessity requirement. After all, as they claim, the NTP account was tailored to meet that requirement (Beall and Restall 2006, p. 40). The trouble here is that the NTP account is just a restatement of the necessity requirement. When we state the necessity requirement (to the effect that the premisses of a valid argument necessitate the conclusion) extensionally and in terms of worlds, as Beall and Restall do, we simply get the NTP account. As such, no justification is thereby conferred on the claim that classical logic meets the necessity requirement: restating the requirement does nothing to show that the requirement is satisfied. An argument is needed where none is given.

Beall and Restall then consider the Tarskian model account of validity (2006, pp. 40–1). They note that in order to satisfy the necessity requirement, there must be a guarantee that, for every argument, if it is possible that its premisses are true and its conclusion false, then there is an abstract model in which the premisses are true and the conclusion false. This is guaranteed only if ‘whenever there is a possibility invalidating an argument, anything done by that possibility can be done by some actual model’ (2006, p. 40), thus wedding possibility with the existence of corresponding models. However, whether there is such a correspondence between possibility and actual models requires argument. And if there could be more abstract objects of the ‘right’ sort than there actually are, some TM-valid arguments are valid only contingently; some possible invalidating cases would not be among the actual Tarskian models.

The problem here is that in order to guarantee that the necessity requirement is met for classical logic, Beall and Restall must show that for every possibility there is a corresponding model. Even if in the case of mathematics the presumption that the correspondence holds is made, this is not enough to guarantee that the assumption holds in general, beyond what are recognized as mathematical possibilities. At
this crucial juncture, however, they defer to what they concede is the mathematical *presumption* that relevant possibilities do not outstrip what is modelled by the actual abstract models, rather than argue for that presumption (2006, pp. 40–1). So, regardless of how the necessity constraint is construed, Beall and Restall have not established that classical logic satisfies the necessity requirement; they have merely asserted that it does.

Having thus assumed that abstract reality is as the classical logician needs it to be, Beall and Restall extend their quantificational formulation of the necessity constraint, by quantifying over different kinds of *cases* as a way of specifying the validity of inferences. This extension is needed, since the background semantic structures for constructive and paraconsistent logics are not only the complete and consistent models upon which classical logicians rely. The background semantics for constructive logics includes incomplete (but still consistent) structures, while the semantics for many paraconsistent logics includes inconsistent (but still complete) structures.

Beall and Restall extend their account as follows. Take ‘cases’ to be whatever structures the various partisan logicians use in the background semantics for their favoured logics. The necessity constraint, more generally construed, is thus the requirement that a relation is one of logical consequence only if in all *cases* in which the premisses are true, so is the conclusion or, equivalently, there is no case in which the premisses are all true and the conclusion false.

This extension, however, creates a tension in Beall and Restall’s pluralist framework, at the level of their metatheory. To see this, consider first a trivial version of logical pluralism—trivial because it requires no substantive philosophical argument to convert one to this kind of pluralism. Everyone can concede the following: were there cases (perhaps *per impossibile*) to be accommodated as this logician insists there are, then all and only the inferences countenanced by that logician are valid. If the only cases worth considering are complete and consistent, then our logics should be classical; if some are incomplete and consistent, then constructive; if some are complete but inconsistent, then paraconsistent. Regardless of how partisan one is regarding one’s preferred modes of reasoning, one can defer to advocates of other logics in this way. Hence, this form of (hypothetical) pluralism is trivial. While it is a very interesting development in the history of the codification of good inference that even though there was a very long and quite revered tradition of maintaining that there was only ‘One True Logic’, there came to be considerations that mathematicians, logicians, and philosophers
took to be sufficiently powerful to warrant the exploration of ways of not only expanding Aristotelian logic into what we now think of as classical logic, but also of exploring logics that might, in different respects, be more adequate than classical logic. Our claim is that (hypothetical) pluralism is trivial only in this sense: a cursory glance through modern encyclopaedias or handbooks of formal logic would demonstrate that various alternatives have been defended as codifications of truth-preserving inference. A tour of a good library, not a philosophical defence, is all that is required.

A substantive logical pluralism, however, requires not merely the assessment that *per impossibile* another logic would have codified a genuine consequence relation; it requires that there be multiple consequence relations; more than one relation satisfies the constraints of necessity, normativity, and formality. The rub is the necessity constraint. For the premisses to necessitate a conclusion is a matter of them doing the right thing in *all* cases. Having recognized that cases may or may not be complete, and that they may or may not be consistent, to do the right thing over all cases is to do the right thing regardless of whether a case is complete or consistent. Consequence relations suited to reasoning about complete situations fail to satisfy the necessity constraint, since by following them exclusively we do not manage our inferences correctly over incomplete cases. Recognizing this is exactly why constructive logicians reject some classically and paraconsistently valid inferences, such as double negation elimination. Similarly, consequence relations suited to reasoning about consistent situations fail to satisfy the necessity constraint, since by following them exclusively we do not manage our inferences correctly over inconsistent cases. Recognizing this is exactly why paraconsistent logicians reject some classically and constructively valid inferences, such as explosion.\(^1\) Thus, on Beall and Restall’s account, none of the major families of logics they consider satisfy the necessity constraint. By their own standard, none of these are logics at all.

Only a very weak consequence relation survives this scrutiny, according to their accounting of the necessity constraint as quantification over all cases. This relation is just the intersection of the inferences treated as valid by classical, constructive, and paraconsistent logics. Some fragments of positive logic and some rules of identity will survive. This survival, however, is merely an artefact of having considered only the cases

\(^1\) A logic is explosive when in that logic anything follows from a contradiction. Reasoning about inconsistent cases and permitting anything to follow from the inconsistency of the case yields triviality, that is, everything is derivable in that logic.
appropriate for the semantic underpinnings of classical, constructive, and paraconsistent logics. If expanding the domain of our metatheoretic quantification to accommodate the semantic underpinnings of constructive and paraconsistent logics is in order, so is the similar expansion to accommodate the semantic underpinnings of non-adjunctive logics, which would invalidate instances of conjunction introduction, and of certain quantum logics, which would invalidate typical laws of identity and distribution. Once the partisan spirit of logical monism is replaced with the open-minded embrace of cases suitable to alternative logics, no commonly promulgated consequence relation seems to satisfy the necessity constraint. Hence, according to their own accounting of the constraints on relations of logical consequence, there are no such relations, much less multiple relations of consequence. Their account of the necessity constraint on logics ends not in logical pluralism, but in logical nihilism.

The source of the problem is the combination of (1) extending what counts as a case for evaluating the validity of an argument form so as to accommodate the background semantics for both classical and non-classical logics, (2) keeping the standard semantics in the metatheory for the quantifier in the necessity constraint, (3) and keeping the metatheory’s domain of quantification fixed. A natural manoeuvre is to shift the domain of quantification when considering each logic. When considering classical or constructive inferences, turn a blind eye toward inconsistent cases; when considering classical or paraconsistent inferences, turn a blind eye toward incomplete cases. There certainly is precedent in ordinary discourse for altering the appropriate domain of quantification as suitable for the conversational context. Thus, ordinary utterances of ‘There is no more milk’ restrict the domain of quantification to one’s own house, for instance, not the entire universe.

This kind of selective attention, however, fails to secure logical pluralism. For the logician, this is nothing more than plugging one’s ears when a logician of different stripes declares some of one’s favoured inferences to be invalid because previously unrecognized cases need to be accommodated. Worse, the strategy of selective attention is a double-edged sword. By ignoring the cases that invalidate classical inferences, we can declare that classical logics satisfy the necessity constraint. Likewise for the other well-known logical systems. If we are even more selective in our attention to invalidating cases, however, any set of inferences can be declared to satisfy the necessity constraint. Pick any pattern of inference you like. Ignore all invalidating cases. Voila! A genuine consequence relation is born! The only inferences that could
not be legitimized in this manner are those with logical falsehoods as their conclusions. Thus, without selective attention, logical nihilism follows. With it, (near) logical universalism follows (according to which virtually every argument is valid). This is a reductio of the strategy of understanding logical pluralism by way of cases.

Perhaps to avoid both nihilism and universalism, Beall and Restall distinguish two ways of endorsing a logic: strong and weak endorsements. One strongly endorses a logic when one includes the actual case among those cases across which one evaluates inferences. If the actual case is not so included, one merely weakly endorses a logic (Beall and Restall 2006, pp. 82–3). While it may be that advocates of different logics characterize the actual case differently, the focus on the actual case is a red herring. Endorsing an historical work requires that one judge that it correctly characterizes the appropriate portions of the actual case. While endorsing a logic does concern whether the sanctioned inferences permit the transition from actually true premisses to an actually false conclusion, the actual world is hardly relevant to endorsing a logic, since the inferences must never, in any case, permit a transition to falsehood from truth. Excluding merely the actual case from consideration when assessing a logic would be useful were the actual case the only case to invalidate certain inferences of a given logic. Unless cases are treated very unorthodoxly, however, regardless of how one characterizes the actual case, there are many cases that invalidate the relevant inferences. Furthermore, excluding the actual case is no help for the classical logician hoping to embrace standard non-classical logics in some manner weaker than monistic advocacy. Those logics include many cases inappropriate for the assessment of classical logic, since they invalidate some classical inferences. It is not as though the semantic underpinnings of other logics differ from those of classical logic regarding the (in)complete or (in)consistent nature of the actual case. They each involve many such cases. So, even if a partisan classical logician were to refuse to assess logics with respect to the actual case, there would remain infinitely many incomplete or inconsistent cases over which the metatheoretic quantifier in the necessity constraint ranges. The verdict, then, is still that classical consequence fails to satisfy the necessity constraint and, hence, it is not an admissible consequence relation. Similarly, for constructive and paraconsistent consequence. Pluralism by cases ends in nihilism even after the distinction between weak and strong endorsement is accommodated. Beall and Restall provide grounds for thinking that there is no ‘One True Logic’, but at the cost of there being no logics at all.
Beall and Restall’s characterization of weak endorsement is ambiguous, however. Perhaps their point is not that strong or weak endorsement is a matter of maintaining that the actual case is among the cases over which the metatheoretic quantifier ranges; perhaps the issue is whether the allegedly invalidating cases for a given logic are taken to be actual cases. This reading avoids the criticism above. It does not, however, evade the fundamental problem with the necessity of valid inference as characterized by cases. Recall that when discussing whether classical logic satisfied the necessity constraint, they ultimately made their case in terms of Tarskian models, and adopted the presumption that for all (potentially) invalidating cases, there was some mathematical object or other possessing the right structure that serves as a model of the premisses and the negation of the conclusion for each invalid argument. Let us grant this model-theoretic claim.

By way of illustration, suppose that one thinks that there are incomplete cases as the constructive logician urges us to think. Since the entire point of invoking the mathematics of model theory is to have actual objects to serve as actual invalidators of invalid arguments so that quantification over cases could do duty for the modality in alternative formulations of the necessity constraint, this commits us to thinking that incomplete cases are among the actual cases. If so, then on this construal of weak endorsement that we offer to Beall and Restall, there is no space to be a pluralist about both classical and constructive logics, for example.

To find a rationale for taking constructive logics seriously in the way Beall and Restall do involves countenancing precisely the cases that the constructive logician claims to be invalidators of some classical inferences. Since the move to model theory is exactly the move to treat the relevant mathematical objects as actual, there is no longer any scope to treat the constructive logicians’ distinctive cases as merely possible cases that might have existed, but actually do not. The relevant mathematical objects either exist or they do not. If they exist, then they are among the actual invalidators for some inference, including some classically sanctioned inferences; recognizing this requires one to reject classical logic in favour of some constructive logic. If they do not exist, then there are none of the distinctively constructivist invalidators of classical inferences and there is no point at all in foreswearing the full power of classical logic, since there are no invalidating cases for any of those inferences. On this alternative reading, then, there are no resources for saving an understanding of the necessity constraint in terms of quantification over cases. No other construals of weak
endorsement seem to be available, so we conclude that accounting for weak endorsement is insufficient to rehabilitate logical pluralism in terms of cases.

This discussion of weak endorsement shows, as well, that characterizing the necessity constraint by quantifying over a domain of cases leaves one without resources to characterize the normativity constraint in a useful manner as well. The intuitive idea is that inferences involving invalid argument patterns involve an inferential mistake rather than merely an inferential preference. Anyone recognizing a domain of cases sufficient for constructive or paraconsistent logics, though, has every ground for judging that classical logicians err when they reason in a manner invalidated by incomplete or inconsistent cases. There just are cases in which the premisses of those arguments are true and the conclusion false. Turning a blind eye to how these inference patterns are invalidated by some cases is no less to ignore the inferential mistake than it is to ignore the mistake in moving from a disjunction to its first disjunct. Of course, if we are so moved, we may ignore all cases in which the disjunction is true, but its first disjunct is false, and treat the inference as valid. We would just be in error were we to do so.

Our error, however, is not a violation of Beall and Restall’s three constraints on an admissible consequence relation. Necessity is satisfied since we are using the ‘blind eye’ strategy to avoid confronting absolutely all cases countenanced by more standard logicians, thus ruling out of our evaluative context all invalidating contexts. Given this selection of cases, normativity is also satisfied, since it would be a mistake to infer the negation of the first disjunct, given that in all of the selected cases the first disjunct holds. For the same reason, it would be a mistake to refuse to infer the first disjunct, given the disjunction. Finally, formality is satisfied as well, since the value of the inference does not depend on the identities of any individuals. Pluralism by cases, then, can accommodate neither the necessity nor the normativity constraints appropriate for an admissible relation of logical consequence, unless the blind eye strategy is adopted.²

Perhaps Beall and Restall have an even weaker form of logical pluralism in mind. They consider the Generalized Tarski Thesis, according to which: ‘An argument is valid, if, and only if, in every case, in which the premisses are true, so is the conclusion’ (Beall and Restall 2006, p. 29).

²In other words, the necessity constraint can be met too easily by a judicious selection of models. Any set of premisses that are linked with a conclusion by way of a material conditional will meet the necessity constraint simply by taking the classical models to be all and only those in which not all the premisses are true or the conclusion is true. The procedure here is to choose the models in such a way that the result will come out right. The necessity constraint is now vacuous.
By changing the specification of cases, and by arguing that the resulting consequence relations are admissible—since they meet all the criteria above—different logics emerge. So, if we quantify over consistent and complete cases (Tarskian models), we obtain classical logic. If we quantify over consistent and incomplete situations, we obtain constructive logic. If we quantify over inconsistent and complete situations, we obtain paraconsistent logic. According to the cases considered by each type of logician, the resulting consequence relation is admissible, and deserves to be taken as a logic. Since more than one logic is thereby obtained, we have logical pluralism. End of the story.

The trouble with this way of characterizing logical pluralism is two-fold: it either overgenerates or undergenerates logics. It overgenerates, if we are permitted to simply choose whatever cases we want to quantify over. For example, select as the cases precisely those situations containing only one object. This selection of cases validates the inference: something exists, therefore only one object exists. Note that all of the requirements imposed by Beall and Restall are now met. First, as for necessity, given the choice of cases, it is necessarily true that if something exists, then only one object exists. Second, as for normativity, it would be a mistake to infer the existence of more than one object or the existence of none, since that would be moving from a true premiss to a false conclusion, given the selection of cases. Finally, as for formality, clearly the inference does not depend on the identity of the objects under consideration. But it is clearly inadequate to consider such inference logically valid on the basis of this peculiar choice of cases!

Alternatively, this account of logical pluralism undergenerates logics. After all, suppose that after seeing the overgeneration, the logical pluralist insists that we should quantify over all cases—rather than over all cases of a certain sort (complete and consistent situations, incomplete situations, etc.). If we quantify over all cases, then nothing that counts as a proper logic will be recaptured. Inconsistent and complete situations will invalidate classical inferences; incomplete and consistent situations will invalidate paraconsistent inferences; complete and inconsistent situation will invalidate constructive inferences. If anything survives such a massive strike, it is unclear that it deserves to be considered a logic. In pursuing the dialectical options open to Beall and Restall, we are back to our initial criticism of their metatheoretic quantificational strategy.

We have argued that making pluralism a matter of cases plus the requisite ‘blind eye’ strategy makes it difficult to characterize the normativity constraint in a useful manner. Making logical pluralism a matter
of cases also undermines one key argumentative strategy employed by Beall and Restall. Thinking that they have already established that classical logics satisfy the necessity constraint, they argue that intuitionistic logics do likewise. Their argument is that since ‘any Tarski model is a stage model of intuitionistic logic where there is only one stage’, and thus ‘no possibility is ruled out by the requirements of intuitionistic logic’, the latter meets the necessity requirement, given that classical logic does (2006, p. 69).

Since this is a ‘piggyback’ argument, if classical logic fails to meet the necessity requirement, this argument for the necessity of intuitionistic reasoning crumbles quickly. Such a result is inevitable, once one considers the motivations for developing non-classical logics. Classical logicians assume certain things that others reject. Intuitionists reason that were all cases complete and consistent, then classical logics would codify valid inference. Intuitionists fail to see sufficient grounds for maintaining that all cases are complete, so the classical grounds for maintaining all of the classical instances of logical consequence likewise fail. The very point of developing non-classical logics is the supposed recognition that some classical inferences fail the necessity test, and they do so because they fail on the cases of peculiar interest to the intuitionist. Once those cases are admitted as those over which reasoning should behave properly, the old arguments proving that classical logics satisfy the necessity constraint are plainly inadequate. Any such proof would have concerned only whether classical inferences behave properly over the complete and consistent cases. Once incomplete cases are deemed admissible for the evaluation of inferences, the standard metatheoretic arguments for the necessity of classical consequence fail to even so much as concern themselves with the complete domain of cases. The arguments do not even deal with incomplete cases, so prior to a proof that any classical inference behaves properly over incomplete cases, no grounds have been provided for thinking that they do, which is precisely what the intuitionist requires for a proof of the necessity of intuitionistic consequence.

The piggyback argument assumes what no classical logician ever provided: an argument for thinking that any classical inferences behave properly over incomplete cases. Indeed, Beall and Restall’s argument is exactly backwards. That Tarski models are special cases of the intuitionist’s stage models shows only that no Tarski models will invalidate intuitionistic inferences. Intuitionism satisfies the necessity constraint, were we to confine out attention to Tarski models. No one, however, maintains that Tarski models invalidate intuitionistic inferences. It is rather
that some stage models invalidate some classical inferences. Implicit in the quotation above is the recognition that the intuitionist is concerned with a larger space of cases and it requires a new, independent proof that either classical or intuitionist logics satisfy the necessity constraint over this larger space of cases. So, piggybacking on the alleged virtues of classical logic is not a useful strategy for the pluralist. Certainly, intuitionists are free to produce arguments showing that their favoured arguments are not invalidated by any consistent cases, whether complete or incomplete, and they do. Such arguments are no help to the logical pluralist, since they are not offered as an indirect defence of classical logic, but rather as supporting monism about intuitionistic logic.

3. From models to modality: modalist logical pluralism

Beall and Restall’s approach to logical pluralism generalizes orthodox approaches to logic. They try to capture the intuitive idea that valid inferences have modal import with the necessity constraint. They, then, explain that constraint in various ways, all of which involve quantification over some domain. It is precisely the attempt to extend this quantificational account, so that a variety of logics satisfy it, which led to the problems of the previous section. Closely aligned with the generality of this quantificational approach is the quite natural tendency to discuss the important features of logical consequence metalinguistically. Neither the typical quantificational nor the ascent into a metalanguage is mandatory, however.

The point of quantifying over a domain of cases is to capture the idea that there is a logical, that is, necessary, connection between the premises and conclusion of valid arguments. The quantification made it difficult to express this necessity in a way that permits logical pluralism. In order to guarantee that because something holds in every case it holds necessarily, we need to ensure not only that it indeed holds in all cases, but also that it holds in all possible cases. To meet the necessity requirement, it is not good enough for ‘all cases’ to be simply all cases; ‘all cases’ need to be all possible cases. If the space of cases is not shown to exhaust all possible cases, no claim to satisfying the necessity constraint is warranted for any proposed logic. To articulate what it is to satisfy the necessity requirement we need some background modal notion. Instead of quantifying over cases, we should rather embrace as primitive the modal notion that the quantification over cases was intended to express. The modal notion is taken as primitive, since any attempt to define it in non-modal terms is subject to the same argu-
ment we just provided regarding all the cases. This embrace of some suitable modal notion as primitive is the position we take to be modalism.\(^3\) We follow Shalkowski (2004) and Vaidya (2006) in reviving what had been, prior to the development of contemporary model theory, the standard way of thinking about one of the key features of valid inference. An argument is valid if, and only if, the conjunction of its premises with the negation of its conclusion is impossible. In addition to the arguments advanced by those authors, we take the arguments of the previous section to warrant a move away from the model-theoretic approach to understanding the fundamentals of logic.\(^4\)

Treating a modality as primitive and as more fundamental than cases has the advantage, in the context where the most significant issue is the characterization of logical consequence, that consequence can be characterized without presupposing a logical notion in our metatheory, that is, the universal quantifier. Let us presuppose a notion of possibility. When considering inferences, and when considering what recommends some over others, we recognize that in an argument such as:

\[
(1) \text{Snow is white} \\
\text{Therefore, (2) something is white}
\]

it is not possible that snow be white while nothing at all is white. It is that quite basic judgement of impossibility that is one of the key features of the argument that recommends it over others such as:

\[
(1) \text{Snow is white} \\
\text{Therefore, (2) something is green}
\]

In other words, it is easy to judge correctly that it is possible for snow to be white while nothing is green. Hue-challenged worlds are not impossible. Thus, snow’s being white guarantees that something is white, and that guarantee is part of what recommends the one argument over the other.

Such (im)possibility judgements are adequate to formulate in nonquantificational form the necessity requirements for logical consequence: \(B\) is a consequence of \(A\) if, and only if, the conjunction of \(A\) and \(\text{not-}B\) is impossible. With consequence, the modalist then obtains the

\(^3\) Strictly speaking, the modal notion need not be primitive in one respect: \emph{when it is contrasted with certain essentialist notions such as what it is to be a (kind of) thing}. We treat the modal notion as primitive here, but no part of our case depends on the rejection of serious essentialism, which takes matters of essence or the identity of an object to be even more fundamental than modality.

\(^4\) Note that we take issue not with whether cases are treated as primitive (they are not), but with the implicit modal significance of the necessity constraint being treated in a way that takes quantification over cases to do the duty of the required modality.
notion of consistency: \( B \) is consistent if, and only if, \( B \) does not entail a contradiction (i.e. a statement of the form \( A \) and not-\( A \)). Thinking of things in this order—making the modal intellectually more basic than logic as a systematized theory of proper inference—has the advantage of making sense of the warranted attention that logical matters receive. First, we have judgements of (im)possibility, which we regiment and codify into the syntax of a logically well-behaved language. Were the syntax not constructed to encode some modal information, the syntactic form of a claim would either be of little interest when trying to think about matters modal, or else it would be an utter miracle that the syntax for expressing claims could bear any interesting relations to the (im)possibility of those claims. At least these are the only options on any kind of naturalistic account of the development of human cognition and language.

A second advantage of modalist accounts of at least the necessity constraint on logic is that it permits us, when being precise about the theoretical fundamentals with which we work, to remain in the object language and to resist the temptation to ascend to a metalanguage and rely upon a truth predicate. Remaining in the object language has two advantages. First, it keeps before our minds more clearly that truth bearers are tools of communication and, as tools created by communicators, their primary function is to convey information about matters outside of themselves. The sentence 'Snow is white' is useful primarily because with it we convey that snow is white. It is less interesting as an object in its own right, though of course there are plenty of intriguing issues about meaning, reference, and the like. Our goal is not to delimit arbitrarily the scope of interesting philosophical problems on which one might work fruitfully. Our goal is, rather, to avoid philosophical distraction when thinking about the nature of logic.

The second advantage that remaining in the object language provides one defending logical pluralism is that it permits the pluralist to characterize the appropriateness of distinct logics in terms of subject matters rather than cases. The logic most appropriate to use in any given circumstance is determined by what one reasonably takes the modally relevant nature of the subjects of concern. We illustrate this below.

As noted, in standard contexts there is a natural link between impossibility and inconsistency. When considering paraconsistent logics, however, this link is inappropriate. Two impossibility judgements are operating for typical classical logicians. First, there is the judgement that it is not possible that all members of an inconsistent set of claims be true. In other words, if claims are composable, then they are consist-
Second, there is the judgement that what follows classically from any inconsistent case—that is, that everything is the case—is impossible. Expressed in terms of theories, the trivial theory follows from an inconsistent theory. Paraconsistentists have tried to provide frameworks within which inference can be managed even when reasoning about some inconsistencies. Managing inferences in those contexts requires that not everything follow from a contradiction. Thus, the most general way to construe possibility, for the purposes of expressing logical pluralism, is in terms of the non-trivial. A theory is immediately trivial when it is every claim; a theory is indirectly trivial when it entails every claim. A possibility, then, is one that is not even indirectly trivial.\footnote{On the condition that the result is a relation of logical consequence that does not validate every inference, some cases must be excluded, at the very least the trivial case. (Of course, in such a case, every relation of logical consequence is admissible, since everything is true. The kind of relation of logical consequence we are interested in is one that permits us to make a serious discrimination between valid and invalid inferences, which is not possible in the trivial case.)}

This appears to have the very counterintuitive result that at least some contradictions are possible. The counterintuitiveness of this arises from an insufficient attention to detail. For all who maintain that no contradiction accurately characterizes the world, the attraction of classical logics is that these quite clearly encode this rather mainstream thought about the essential character of the world. When we cease to focus exclusively on what the world makes true, though, it is perfectly obvious that contradictions are possible. Note: possible, not possibly true. In fact, we are all quite familiar with the fact that contradictions abound. Philosophical works, student essays, political speeches, and the beliefs of everyone who embodies sufficient intellectual humility to recognize that some of their own beliefs are incorrect all contain contradictions.

This attention to what should be a small detail opens an avenue for logical pluralists. Though there are plenty of bones of philosophical contention scattered around discussions of metaphysics, alternative logics, and their relations, we can remain quite neutral on these matters. Our arguments are informed by what we take to be orthodoxy not because we believe that there is some powerful presumption in favour of orthodoxies, but because we wish to make our argument in terms of examples and we wish the examples to be as uncontroversial as possible. The main points that pertain to pluralism can be maintained even for those who resist some of the orthodoxies toward which we gesture.

Suppose, then, that one thinks that the world is both consistent and complete, and by that we mean only that nothing both is and is not the
case, and that every matter upon which it is possible to claim, the matter is settled one way or the other.⁶ If our subject matter is the character of the world, or even the character of the way any world could be, then it would be quite natural to use the full range of classical inferences when reasoning about the world.⁷ Note that our subject matter here is the character of the world. There certainly are times when our interest is merely empirical. We care not how the world as a whole might have been; we care only whether there is a barn in the field, or whether the weakness of gravity can be explained by way of the existence of a multiverse. Even when we are interested primarily in straightforwardly empirical matters, we often use inferences.

Now what is the point of using inferences whether in the context of empirical investigation or not? The main point of inference is that it permits us to extend our knowledge, in the sense that it helps us realize some of the consequences of things we already know. Given what we know to be the case, we can determine further things to be the case by determining the logical consequences of what we know to be the case. Furthermore, we can determine what is not the case by determining what is incompatible with what we know to be the case.

Since we often make our knowledge explicit without the need for further observations on our parts, the following principle recommends itself: as appropriate, use all of the tools that permit one to extend one’s knowledge over a given domain without error. Since we have, for the sake of argument, joined the orthodoxy in taking the world to be consistent and complete, it would be foolish needlessly to fail to use all classically valid inferences as appropriate. There is no point in tying one’s hands behind one’s own back.

⁶ In line with standard presentations of classical logics, we simplify things to favour the legitimacy of classical logic. For instance, we ignore the perfectly obvious phenomenon of vagueness and we do so partly because some hold that vagueness can be handled perfectly well without any alterations to classical logic. Epistemicism regarding vagueness is a clear example of this. If troublesome things like vagueness were merely a matter of imprecision in the semantics for descriptive terms, or if vagueness reflects more the state of our ignorance than it does the state of the world, then a language and a system of inference that ignored vagueness is not inaccurate regarding the character of the world in itself. (For the purposes of this paper, we leave aside questions of the legitimacy of the idea of the world in itself.)

⁷ For those who worry about some inference from the consistent and complete character of our world to the consistent and complete character of any world consider how it is that anyone other than God could come to be warranted in believing that our own world is both consistent and complete. It is not by inspection, since we are able to do far too little of that to warrant on empirical grounds these claims about our world. In so far as anyone so much as has grounds for these claims those grounds would be a priori grounds to do with the very nature of a world, any world, of which ours is but one. So, it is implausible to see those maintaining reigning orthodoxies about the character of the world as making some dubious generalization based on a single instance.
Suppose, though, that one is interested not in reasoning about the consistent and complete world, but in reasoning about claims made about the world, such as which students are enrolled in a given university class, what their respective student identification numbers are, how many credits each has earned, how many library books loaned to a particular student are overdue, etc. Those who have managed large databases know only too well that insuring consistency of data, that is, maintaining the mutual consistency of all of the claims represented in the database, is not a trivial matter. Not surprisingly, databases are sometimes inconsistent. In one place a student is represented as having three overdue library books when the library’s catalogue shows them available for borrowing and on their proper shelves. When the student tries to finalize the processes that lead to graduation, the database prevents a university administrator from issuing that student’s graduation certificate because the program controlling that portion of the process still contains the information that the student has failed to return all of the library’s property that has been borrowed.

The world is not inconsistent, since the books are where they are and not somewhere else and the student has done what the student has done and not something else. There is no impossibility in claiming both that the student has returned the books and also that the student has not. So, while the world is perfectly consistent, claims about it need not be. Suppose now that we want to reason not about the world. We already know that the books have either been returned or not. We do

8 The contrast between the world and claims about the world is often obscured by the facile reference to premises and conclusions in arguments. When we put up on the chalk board for students to see:

\[
(1) \text{ Snow is white}
\]

and we ask whether it is true or whether it is the case, what is the ‘it’? Is it an object to which ‘is true’ might be an appropriate predicate? Is it, instead, an object to which the predicate is not so appropriate, such as a fact or a state of affairs? Do we mean to make some reference to the world, but not to a distinctive ontological unit that is the peculiar interest of ontologist, but to ordinary speakers before they become acquainted with the niceties of metaphysics? Our modalist position, which stays in the object language whenever possible when being philosophically serious, involves nothing that should strike anyone as dubious metaphysics. Talk of truth, truth bearers, truthmakers, facts, etc. is talk that is designed to permit us to speak generically about things in a highly abstract manner. Speaking of when the premises are true and the like carries no deep ontological insight to propositions, for instance. It merely permits us to talk about not merely this particular argument and that particular argument and what follows when those particular premises are true; it permits us to talk about any argument, perhaps of different forms. All of this metalinguistic talk, however, is simply to systematize quite general features of specific arguments and those arguments have many of the features they do in virtue of what they are about, be it snow, grass, or anything else. It is, after all, only in virtue of the things that are part of the world having certain characters that we can infer about the characters of other objects or about the character of the world. The general character of truth bearers is philosophically interesting only in so far as they encode something informative about their respective contents.
not, however, always know which. In the case described, there might be an easy way of resolving the empirical matter of whether the books are currently on their proper shelves, even if there may be no easy way to get the technology to let us print the student’s diploma in time for the graduation ceremony. But perhaps the empirical matter cannot be so easily resolved. In which case, we will need to reason about these inconsistent claims, and in order to do that, it needs to be possible for the inconsistent claims not to trivialize our whole database. In other words, it makes perfect sense, in this case, to adopt a logic in which the inconsistency of the claims under consideration does not entail everything. It makes sense to use a paraconsistent logic.

The paraconsistent logician effectively expands the domain of the possible. By allowing that certain inconsistencies might lurk among claims, the paraconsistent logician is in a position to reason about an inconsistent domain without the triviality that classical logic would impose. There are, of course, several non-equivalent paraconsistent logics, and several of them would be adequate for the task. While this is already a limited logical pluralism, it is not logical relativism in the sense that any old logic would be adequate for the task of reasoning about inconsistent databases. Classical and intuitionist logics, for example, given that they sanction the validity of explosion, are singularly inappropriate.

Expanding the way we think about the domain of the possible is what the non-classical logician typically does and this is perfectly in line with modalism. There are different ways of expanding the domain of the possible: by entertaining inconsistent situations, by considering incomplete situations such as all and only the contents of one’s office, by allowing for scenarios in which the notion of identity cannot be applied to certain things (such as some cases considered in certain interpretations of quantum mechanics). By expanding and exploring the possible in this way, the modalist can identify contexts in which the validity of certain classical inferences is violated. Logics that are more restrictive, that is, those that deem fewer patterns of inference to be truth preserving, emerge. These alternative logics are better suited than classical logic to reasoning about the objects under consideration. And, once again, often several such logics would be adequate to reason about the objects in a given domain. As a result, modalism and logical pluralism go hand in hand.

It should be clear that admitting the possibility of inconsistent claims, or of incomplete situations, or of objects to which identity does not apply, involves nothing metaphysically suspicious. Modalists, considering the possibility of inconsistent claims, are not assuming that the
world is inconsistent. They simply consider the conjunction of inconsistent claims. Similarly, when considering incomplete situations, one is simply considering certain parts of the world, not the world as a whole. Again, when one considers the description of objects to which identity does not apply, one simply considers a particular description of such objects, based on a particular interpretation of quantum mechanics. No commitment to the existence of such objects is needed. Thus ‘expanding the domain of the possible’ is not incoherent or metaphysically dubious. The possible is what it is independently of us. We cannot ‘expand’ that—any more than we can change the laws of nature. ‘Expanding the domain of the possible’ is simply a figurative way of describing what non-classical logicians do when they entertain possibilities that are precluded by fiat by classical logic. These are possibilities that are taken to be genuine by the non-classical logician. Indeed, many of them are widely recognized to be parts of the actual world.

Having made the case that the resources to which non-classical logicians draw our attention are often perfectly ordinary things, we begin to set forth the distinctive character of our account of logical pluralism. Having distinguished complete and consistent worlds from incomplete but consistent situations from potentially incomplete and inconsistent databases, focus not on the background semantic structures for the various logics under consideration, that is, do not focus on the various types of cases entertained by Beall and Restall. Focus instead on the kinds of things we reason about. After all, only in highly theoretical contexts do we reason about the background semantic structures for various logics. Call these different kinds of things that we reason about ‘domains’. Different domains are just different subject matters along with various ways of conceptualizing those subject matters. Different domains or subject matters typically concern different objects.

Since they typically concern different objects, different subject matters routinely call for different ways of thinking. Granted, the most ordinary idea to have here is to think that different subject matters might require that our theories of those subject matters be framed in terms of different laws. As far as this goes, this is perfectly permissible. It is also plausible to think that because of the different character of the objects of concern in the various subjects, these objects might well warrant different canons of inference when reasoning about them.

Consider once again databases that might well be inconsistent. Assuming, for example, classical or intuitionist logic, any automated reasoning about a database that turns out to be inconsistent would, ultimately, break down. A clever automated reasoner would conclude
the trivial theory as soon as an inconsistency is spotted. The classical verdict is that every inconsistent theory is implicitly trivial. Those of us who are not so simple-mindedly automated, though, are perfectly capable of compartmentalizing that inconsistency and proceeding to reason about the rest of the information, or of reasoning hypothetically about various consistent subsets of the claims in the database, which might involve some members of the inconsistent subset of the database. For compartmentalization to be useful, however, a reasoner must, in effect, shift away from thinking about the entire inconsistent database and consider, instead, different databases, in which the inconsistency is removed. Well-managed inference permits us to deduce what follows from some assumptions and, implicitly, rule out things that are not the case, if the assumptions hold.

This is precisely what cannot be done when reasoning classically about an inconsistent database. Hold fixed the entire inconsistent database; select a consistent subset of that database; make classically valid inferences from the consistent subset. It is correct to think that we can, at least, conclude that those inferences would hold were the entire database just that particular consistent subset. Since it is not, classical inferences are useless in extending our knowledge about the original database, since—classically speaking—the negation of every classically derived conclusion from the consistent subset can be derived from the original. In the context of inconsistencies, classical reasoning does not permit us to use inference to rule things out as not the case. Given the classical validity of explosion, more sophisticated reasoners have no advantage over a simple-minded automated reasoner that scans for inconsistency before all else, and then shuts down if inconsistency is discovered or derived. For this reason, paraconsistent logics that provide guidance for reasoning about inconsistent assumptions were developed. When thinking about databases, paraconsistent consequence is appropriate to use, and even those who insist that no world could be inconsistent benefit from reasoning paraconsistently when confronting databases.9 Attempting to reason classically in that domain is either not to take classical logic seriously or else it is, strictly speaking, to give up on a key point of having a system of well-managed inferences.

9 As noted above, there are many (in fact, infinitely many) paraconsistent logics (see da Costa, Krause, and Bueno 2007). If one of them is appropriate for a given domain, several other paraconsistent logics will be equally suited. This is not unique to paraconsistent logics. We have logical pluralism even within a domain.
Suppose that instead of databases we wish to think about constructive mathematics or other contexts in which an assertion is legitimate only when it is asserted with sufficient evidence, where the relevant evidence must not arise from some global claim such as something of the form: A or not-A. When thinking about proofs, one might not be in a position to assert not-A, from which it does not follow that one is in a position to assert A. At most stages of the baseball season one might, in quite ordinary contexts, say: ‘I am not certain that the Yankees will not win the World Series’. Quite clearly, suppose that a hearer used a simple-minded version of the classically sanctioned double negation elimination to infer that the speaker has, effectively, claimed to be certain that the Yankees would, indeed, win the World Series. The hearer would be guilty of not only conversational but also inferential error. In the classical context, the tendency is to simply declare that the sentence in question, an appearance to the contrary, does not really have the form of ¬¬p, thus making double negation elimination inappropriate. The entire rationale for declaring that this ordinary sentence fails to have this form even though there are clearly two instances of ‘not’ in the sentence is that treating the sentence as one having that form entails that classical inferences will sometimes lead to false conclusions from true premisses. So, we declare that when intensional contexts are in question, sometimes what appear to be claims with a specific discernible logical structure do not really have that structure. Thus, the above is treated as a simple negation with the unnegated sentence being ‘I am certain that the Yankees will not win the World Series’. This exclusion on this basis and any attending ways of handling the formal representation of claims involving intensional contexts is just a tacit admission that classical inference is not suitable for thinking about intensional contexts, including degrees of doxastic certainty or epistemic warrant.

Thus, when thinking about at least some epistemic states, some version of constructive logic is more appropriate than is classical logic. As in the case of making room for the occasional appropriateness of para-consistent consequence, the occasional appropriateness of some constructive consequence requires no concession to one who might hold that reality is, or at least could be, incomplete. One who can make absolutely no sense of reality—this world or any (possible) other—being incomplete can nevertheless make room for the use of constructive logics as appropriate when thinking about some domains, like that of some cognitive states.

The reasons for the occasional appropriateness of non-classical consequence relations highlight why focusing on the details of the back-
ground semantic structures for various logics (i.e. the different kinds of cases that Beall and Restall consider) is unhelpful. That valid inferences involve some kind of guarantee for the conclusion, given the premises, is what motivates making necessitation a constraint on a proper consequence relation. When attempting to model inference patterns in purely extensional, existential terms—classically, in terms of existing abstract models—the modelling goes haywire as soon as new models, or situations, or cases are said to exist and to be appropriate to the evaluation of some logical inferences. First, of course, is the concern that the ‘new’ cases are not really possibilities and, given the concern for logical necessitation, inappropriate for evaluating inference. One swayed by this concern will not be persuaded of any pluralism about inference, but of the monism typically associated with the narrower class of cases, typically classical logic. Second, once the domain of cases is expanded to include more legitimate cases for the evaluation of arguments, some inferences previously thought to be valid will turn out to be invalid. Once again, logical pluralism is not warranted, but monism about some different logic.

Focusing on domains of concern as we urge frees us from thinking that the background semantic structures are fundamental to our thinking about logic—both our pre-philosophical and our philosophical thinking—because they are the objects to which all verdicts on consequence are responsible. Those structures are not our route into the proper inferences to draw, except in so far as they are an articulation of the appropriateness of various inferences in various contexts about various things. Focusing on domains rather than on cases puts the inferential horse before the modelling cart, just where it should be.

Focusing on domains also has the advantage of yielding neither logical nihilism nor universalism. Beall and Restall run together two different projects that are better kept apart. One is the characterization of logic (in terms of the three constraints that a relation of logical consequence is supposed to meet); the other is a defence of logical pluralism (by specifying different cases to which distinct logics are accountable). Central to their logical pluralist strategy is a quantificational account of validity. We argued that if the quantification over cases is left unrestricted, we obtain logical nihilism and if it is restricted, the result is logical universalism—almost everything would count as an admissible consequence relation.

Since the modalist logical pluralist is not quantifying over cases, there is no issue of having restricted or unrestricted quantification. The modalist relies only on what is uncontroversial, such as the character of
databases and intensional contexts. The focus is directly on the necessitation of the conclusion by the premises, but this is done in a way that is sensitive to the subject matter (the domain) about which we reason. The resources that the logician uses to model the various patterns of inference should not be conflated with the modality for which they are often asked to do duty.

The fundamental work of warranting pluralism, for a modalist, are judgements about possibility, and these judgements are the basis for determining which arguments are valid or invalid. Those very judgements involve discriminating in familiar ways. Some inferences are judged valid; others are not. Both extremes of logical nihilism and universalism are avoided in a straightforward manner. Logical universalism is avoided precisely because some systems of inference fail straightforwardly to satisfy the necessity constraint, even if they preserve truth as things are, as in the inference that grass is green from the premise that snow is white. As a result, not every system of inference counts as a logic. Logical nihilism is avoided because there are domains about which we reason for which several logics are ideally suited (for instance, different paraconsistent logics are adequate for the same inconsistent domain), and for different domains, distinct logics are adequate (e.g. classical logic for consistent and complete domains, and constructive logics for consistent and incomplete domains).

Perhaps some non-classical logicians may insist that the most generally applicable system of inference is to be preferred in a monistic rather than pluralist spirit. The pluralist resists the suggestion because it involves self-consciously and unnecessarily refusing to make perfectly valid inferences in some contexts. We might know well that the domain over which we are currently reasoning is not a domain over which, for example, relevantists (or other paraconsistentists or constructivists) charge classical inferences with invalidity. Simply because inferences fail in the contexts that relevantists highlight offers no reason to foreshow these inferences in contexts where they are perfectly adequate, and about which non-classical logicians raise no objection to classical inferences. Given the point of engaging in inference in the first place, a demand for universal applicability of every admissible form of argument is an epistemically limiting strategy when inference is, in part, to be an epistemically liberating strategy.

One final issue is whether the position we have defended should be viewed as a genuine logical pluralism or a mere contextualism. We begin by considering what Beall and Restall had in mind, or at least should have had in mind if they did not, regarding pluralism. Recall
that it is completely uncontroversial that there are distinct logical syst-
ems that yield conflicting verdicts regarding the validity of argument 
forms such as double negation elimination or explosion. Cursory atten-
tion to the contents of a good handbook on formal logic (e.g. Gabbay 
and Guenthner 2001) would have revealed this historical fact and no 
philosophical defence of this claim was necessary. Likewise, there was 
no need for a defence of the position that by their own lights logicians 
within each distinct family of logics get the facts of logical consequence 
correct. Partisan logicians regularly point out that there is the appro-
riate match between Tarskian models and classically-sanctioned infer-
ences; it is not contested whether inference patterns treated as 
constructively valid really do preserve truth over all incomplete situ-
tions, and the like. We find that partisan logicians tacitly grant that if 
the cases over which truth is to be preserved via inference are as classi-
cal logicians say Tarskian models are, then logics are classical. If the 
appropriate class of cases, though, is that of incomplete situations, then 
the logics are constructive, etc.

The burden of Beall and Restall 2006 is to warrant the thesis that 
there are many correct ways of inferring, not all of which are consistent 
with each other. Now the issue is this: are there different correct ways of 
reasoning merely by the lights of different logicians or are there different 
correct ways of reasoning, full stop? Beall and Restall seem to maintain 
that the former is all that they need to maintain when they say that 
'disjunctive syllogism is valid when GTT’s cases are taken to be possible 
worlds, and it is invalid when those cases are taken to be situations. That is that’ (2006, p. 56). The former, though, is perfectly 
compatible with the logical monist’s position that there is One True 
Logic and that partisan logicians simply disagree over the character of 
that logic, so if the former is the substance of Beall and Restall’s plural-
ism, they defended what required no defence, given the contemporary 
state of the philosophy of logic. So, the latter is what must be at issue for 
logical pluralism to be a philosophically interesting thesis.

One with no firm philosophical commitments about what the space 
of cases contains, that is, one wishing to remain above the fray regard-
ing whether all cases are complete or consistent, can maintain no more 
than the weak position that competent logicians do what is right in 
their own eyes. This has the merit of permitting one to open-mindedly 
affirm that some forms of argument are classically valid and some not, 
some relevantly valid and some not, etc.

As soon as one becomes somewhat more committed regarding what 
cases there are, the defence of the philosophically interesting pluralist
position becomes increasingly unstable, on Beall and Restall’s development of that view. If one thinks that there are complete and consistent cases, such as worlds or Tarskian models, and also that there are incomplete cases, such as situations, one either takes the quantificational account of the necessity constraint seriously or not. If one takes it seriously, then it is impossible to maintain that classical instances of explosion are valid. There are many incomplete situations in which not everything follows from inconsistent assumptions. Consider an inconsistent database containing $p$, $q$, and $\neg q$.\(^{10}\) $\neg p$ follows classically, but not relevantly. Anyone committed to the existence of incomplete situations that they take to warrant the development of relevant logics and for maintaining that relevant logics satisfy the normativity constraint as Beall and Restall do (2006, p. 55), now has grounds for declaring that classical explosion involves a mistake in relevance because the premises are not sufficiently relevant to instances of the conclusion above. There are situations sufficient for the truth of the assumptions, but not for the conclusion, which is just to say that the classical inference does not preserve truth when we account for situations as cases, which is to say that classical explosion fails to satisfy the necessity constraint.

This declaration about $\neg p$, note though, is completely uncontroversial. If we consider only Tarskian models, inferring $\neg p$ is sanctioned; if we widen our scope to consider incomplete situations, it is not. Partisan monists grant this. Pluralists might maintain that it is both correct to infer $\neg p$ classically and correct to refrain from doing so because the information contained in the database is not sufficiently relevant to warrant the inference of $\neg p$, since that conclusion fails to follow from those specific premises as contrasted with other inconsistent premises. If one is attempting to reason about the character the world that one reasonably believes to be consistent, the inference is both truth-preserving and warranted in the context of a reductio argument. We learn that our assumptions are not consistent and we uncover good grounds for re-examining those assumptions in the attempt to rid ourselves of falsehood. When thinking about databases, this may not be the best way to enhance our knowledge via inference since we may well not need to discover that our assumptions are inconsistent. We may already know that, but since we do not want, on that basis alone, to accept the trivial theory or even to use inference at all to determine the location of falsehood since observation might serve that purpose much better, a non-classical logic that treats the above inference pattern as out of bounds is both correct and warranted. Perhaps this position deserves

\(^{10}\) We thank an anonymous reader for the example.
the label ‘contextualism’. Whether it does or not, it certainly deserves the label ‘logical pluralism’, and it is a better candidate for that label, since it permits us a means of articulating and defending the stronger version of pluralism and not merely the philosophically uncontroversial weaker version.

4. Conclusion

The preceding discussion makes it clear that we have four main alternatives in the debate about the range of logic: logical nihilism, monism, pluralism, and universalism. Just as Beall and Restall, we favour the logical pluralist option. As opposed to them, however, we favour a modalist conception of logical pluralism that, by emphasizing domains and a primitive modality, is able to avoid collapsing into the extremes of nihilism and universalism. With modalism in place, a genuine logical pluralism is possible.11

References


11 We are grateful to two anonymous readers for helpful comments.