Empirical factors and structure transference: Returning to the London account

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Abstract

We offer a framework to represent the roles of empirical and theoretical factors in theory construction, and examine a case study to illustrate how the framework can be used to illuminate central features of scientific reasoning. The case study provides an extension of French and Ladyman’s (1997) analysis of Fritz and Heinz London’s model of superconductivity to accommodate the role of the analogy between superconductivity and diamagnetic phenomena in the development of the model between 1935 and 1937. We focus on this case since it allows us to separate the roles of empirical and theoretical factors, and so provides an example of the utility of the approach that we have adopted. We conclude the paper by drawing on the particular framework here developed to address a range of concerns.

1. Introduction

It is a commonplace to say that theory construction involves both empirical and theoretical constraints. How these constraints interact, however, is a much more contested matter. Empirical considerations often limit the scope of possibilities for theoretical elaboration, while the latter typically suggest avenues for further research, particularly since they are usually associated with heuristic principles. In many cases, it is very difficult to map out the contributions of each of these types of consideration. We are interested in particular in the way such experimental results can help drive the move to a new model of the phenomena. As we shall see, the move may be more complex than is generally appreciated and, indeed, we shall suggest that it can be conceptually divided into two distinct stages, which we will call, respectively, the characterization and explanatory stages.

Our goal in this paper is thus twofold: First, we present a formal account of these stages within the framework of the partial structures version of the semantic approach. Second, we explore a particular case study using this framework. The episode in question admits of a separation of the respective roles of the empirical and theoretical factors, and thus provides an example of the utility of the approach that we have adopted. The case study extends French and Ladyman’s analysis of Fritz and Heinz London’s model of superconductivity to cover the role of the crucial diamagnetic analogy in the development of the model between 1935 and 1937 (French and Ladyman, 1997). This feature of the episode has not, to our knowledge, been studied previously and here we see an interesting shift from a ‘macroscopic’ model, dealing with current densities and magnetic field strengths, to a microscopic interpretation, based on the motion of electrons, that, although not able to offer a full theoretical explanation of superconductivity (for reasons that shall become clear), greatly reduced the range of options available. What is particularly interesting are the roles of, first, a crucially important experiment (associated with what came to be known as the Meissner effect), and, second, a fundamental analogy with the theory of diamagnetism. The construction of the London and London model was crucially dependent on this experimental result, which was effectively ‘built into’ the model. However, this move does not render the model ad hoc, since the intention was...
not to explain the relevant phenomenon behind this result, but to characterize it in a way that allows the resources of the theory of electromagnetism to be brought to bear. The manner of characterization was determined, in part, by the analogy with diamagnetism, that was increasingly drawn upon to construct the underlying 'microscopic' model which was presented as a partial explanation.

Thus, what we have here, as we shall see, is not simply a case of filling in the value of a quantity which in the construction of the theory had been left open. Instead, the experimental result ruled out the older model and motivated the construction of a new one. As will become clear, in the context of our case study, the two stages of characterization and explanation correspond to the schema adopted by one of the principal protagonists, Fritz London, in his own reflective view of theory construction. It is perhaps unusual to find theory construction separated so explicitly into stages in this manner, but it very nicely sheds light on the role of empirical factors in theory construction.

However, we shall also examine the fundamental role of the theoretical analogy with diamagnetism and the way in which further structure was imported into the superconducting domain in order to frame the developing microscopic model. Such a transfer of structure can also be nicely accommodated within the partial structures account of theories (see, for example, da Costa and French, 2003) which represents the relationships between such theories in terms of partial isomorphisms and homomorphisms.

2. The characterization-explanation framework

The formal details underpinning our framework have been given elsewhere (see, e.g., da Costa and French, 2003), but we shall summarize them before considering how this framework can accommodate the above two-stage process: A partial structure is a set-theoretic construct $A = \langle D, R_i \rangle_{i \in K}$, where $D$ is a non-empty set and each $R_i$ is a partial relation. A partial relation $R_i$ over $D$ is a relation which is not necessarily defined for all n-tuples of elements of $D$ (see da Costa and French, 1990, p. 255). Each partial relation $R_i$ can be viewed as an ordered triple $\langle R_1, R_2, R_3 \rangle$, where $R_1$, $R_2$, and $R_3$ are mutually disjoint sets, with $R_i \cap R_j \cap R_k = \emptyset$, and such that: $R_1$ is the set of n-tuples that (we take) belong to $R$; $R_2$ is the set of n-tuples that (we take) do not belong to $R$; and $R_3$ is the set of n-tuples for which it is not defined whether they belong or not to $R$.

If we have two partial structures, $A = \langle D, R_0 \rangle_{k \in K}$ and $A' = \langle D', R_0' \rangle_{k \in K}$ (where $R_k$ and $R_k'$ are partial relations as above, so that $R_k = \langle R_{k1}, R_{k2}, R_{k3} \rangle$ and $R_k' = \langle R'_{k1}, R'_{k2}, R'_{k3} \rangle$), then a (partial) function $f$ from $D$ to $D'$ is a partial isomorphism between $A$ and $A'$ if and only if (a) $f$ is a bijective, and (b) for all $x$ and $y$ in $D$, $R_{k1}xy \iff R_{k2}f(x)f(y)$ and $R_{k2}xy \iff R_{k3}f(x)f(y)$ (French and Ladyman, 1999; Bueno, 1997).\footnote{For simplicity, we are considering here only two-place relations. But the definition can, of course, be easily extended to n-place relations.}

Moreover, we say that a (partial) function $f: D \to D'$ is a partial homomorphism from $A$ to $A'$ if and only if for every $x$ and every $y$ in $D$, $R_{k1}xy \iff R_{k2}f(x)f(y)$ and $R_{k2}xy \iff R_{k3}f(x)f(y)$ (Buono, French, and Ladyman, 2002). Again, if $R_k$ and $R_k'$ are empty, we obtain the standard notion of homomorphism as a particular case.

Using this formalism, we can also represent the hierarchy of models—what Suppes called models of data, of instrumentation, of experiment, as well as the kinds of 'intermediate' models we shall be considering here—that take us from the phenomena to the theoretical level (Bueno, 1997):

$$S_k = \langle D_k, R_{k1}, R_{k2}, R_{k3}, \ldots, R_{kn} \rangle$$
$$S_{k-1} = \langle D_{k-1}, R_{(k-1)1}, R_{(k-1)2}, R_{(k-1)3}, \ldots, R_{(k-1)n} \rangle$$
$$\ldots$$
$$S_3 = \langle D_3, R_{31}, R_{32}, R_{33}, \ldots, R_{3n} \rangle$$
$$S_2 = \langle D_2, R_{21}, R_{22}, R_{23}, \ldots, R_{2n} \rangle$$
$$S_1 = \langle D_1, R_{11}, R_{12}, R_{13}, \ldots, R_{1n} \rangle$$

where each $R_k$ is a partial relation of the form $\langle R_{k1}, R_{k2}, R_{k3} \rangle$—with $R_{k1}$ representing the n-tuples that (we know) belong to $R_0$, $R_{k2}$ the ones that (we know) do not belong to $R_0$, and $R_{k3}$ those for which it is not defined whether they belong or not—such that, for every $i$, $1 \leq i \leq k$, $\text{card}(R_{ji}) > \text{card}(R_{j(i+1)})$ (Bueno 1997, p. 601). The partial relations are extended as one goes up the hierarchy, in the sense that at each level, partial relations which were not defined at a lower level come to be defined, with their elements belonging to either $R_1$ or $R_2$.

Now, empirical results can be accommodated within this hierarchy in various ways and as we indicated above, we wish to focus on the way such results can help drive the move to a new model of the phenomenon. Such a move can be represented within our formalism as follows:

$$A = \langle D, R_k \rangle_{k \in K} \rightarrow A = \langle D', R'_k \rangle_{k \in K}$$

where, as we shall see, some of the theoretical relations used to help obtain $A$ may be retained by $A'$. In constructing the new model, various heuristic principles may be drawn upon but, again, we shall be particularly interested in the transfer of structure, that is, where structure in one domain is drawn upon to help determine the relevant structure in another. In transferring such structure, analogies may often play a heuristic role, as will become clear below. Here one may represent the situation with two different models—differing in their respective domains, $D$ and $D'$—and which can be related via partial isomorphisms (or other morphisms):

$$A = \langle D, R_k \rangle_{k \in K} \quad \text{and} \quad A' = \langle D', R'_k \rangle_{k \in K}$$

In this case, the ontology of the models is different, and what matters is the way in which the relations from one model are mapped into the other.

These two factors—the empirical and the transfer of structure—may then work together to contribute to what we call the characterization stage, where a determinate characterization of the relevant phenomenon is constructed. What is involved in such a characterization is the stabilization of a given phenomena such that it is amenable to further theoretical development. The stabilization in question in some cases may be largely empirically driven, and thus what it yields is an empirically adequate representation of the phenomena. But we are particularly interested in cases in which theoretical considerations play a prominent role, and thus as we shall see, it would not be correct to present this as a form of phenomenological model construction. As will also become clear, empirical factors both rule out a given model and help to form its successor. In some cases, they may do so by building the relevant results into the model. In the latter respect, the formation of the new model is typically achieved in combination with the relevant heuristic principles (in the case to be considered below, these principles involve certain analogies between different domains).

Structure transference can then be further drawn upon as we move up the hierarchy to the second, explanatory stage. When the empirical results are 'built into' the new model at the characterization stage, that model obviously cannot be taken to explain the phenomenon. However, as we shall see, the same structure transference (or analogy) that is used to construct the new model...
at the characterization stage can then be drawn upon again to guide the delineation of the relevant explanatory mechanism as part of the construction of a further model in the explanatory stage. This further model itself may not be the final word on the matter—that is, it too may be significantly partial—but may point the way to a more explanatory successor. Thus, through this combination of empirical factors and heuristics involving structure transference motivated by analogies, we effectively move up our hierarchy of structures above.

We shall now present our case study illustrating these various moves.2

3. The analogy ‘in the air’

By the 1930s all attempts to explain the phenomenon of superconductivity had apparently met with failure. The situation was characterized by Fritz London as follows:

It seems that the principal obstacle which stands in the way of understanding this phenomenon is to be sought in its customary macroscopical interpretation as a kind of limiting case of ordinary conductivity. The present theoretical situation may be characterized in such a way that it is rigorously demonstrable that, on the basis of the recognized conceptions of the electron theory of metals, a theory of supraconductivity is impossible—provided that the phenomenon is interpreted in the usual way (London, 1935, p. 24).

This obstacle was overcome that same year by a new experimental result (the Meissner effect) that helped drive the shift to a new ‘macroscopical’ interpretation, developed by London himself with his brother. In their joint 1935 paper, they explicitly set out their understanding in terms of the logical relationships between the core equations involved in the shift from the old, inadequate model, which was based on an analogy with ferromagnetism, to their own, based on the analogy with diamagnetism. Here we shall explain this in detail, emphasizing the guidance provided by the diamagnetic analogy. As French and Ladyman record (1997, p. 383), this analogy had already been noted by Frenkel and was ‘in the air’, as it were. Indeed, Frenkel’s paper is interesting with regard to what we shall be presenting below, since, first of all, he presents a microscopic, electron-based model of superconductivity. Now, although this model was rejected, it helps to demarcate the extent to which the background information that London and London were drawing upon was heavily theoretical. Furthermore, the fundamental reason Frenkel’s model was deficient, is that he did not have the appropriate characterization of superconductivity that the Londons found only following the discovery of the Meissner effect (which involves the expulsion of magnetic flux from a superconductor when it is cooled below the transition temperature). This was the crucial first stage in the construction of an appropriate theory, corresponding to our characterization stage. But now, in terms of what we shall consider below, we have an interesting situation: London and London draw on the diamagnetic analogy in the construction of their ‘macroscopic’ model, prior to moving to a microscopic model, also delineated by the analogy, but the context in which that analogy was initially drawn upon was also theoretically grounded on microscopic considerations, as Frenkel’s work reveals.

Second, Frenkel himself explicitly draws upon the relevant analogy with diamagnetism in constructing his own micro-model (see Frenkel 1933, p. 910). He does so to move from a detailed consideration of the motion of electrons in an atom, to that of a metal in a superconducting state, passing through a series of models, and writes:

With regard to its reaction to an external magnetic field, a metal in the superconducting state must behave like a diamagnetic body with a large negative susceptibility. [...] This is quite equivalent to saying that the interior of such a body will be screened from external magnetic fields by the system of surface currents induced by the latter. (ibid., p. 911; his emphasis).

And he further draws upon this analogy to help visualize (his term) what is going on with a superconductor. It is worth noting that the feature of diamagnetism that is brought across to the superconducting case is the screening from external magnetic fields. As we shall see, this formed an important part of the analogy in the London and London case as well. At the end of his paper, Frenkel expresses the hope that a more exact quantum analysis based on his sketch will help resolve certain deficiencies, just as London does in his work, although for London the resolution must begin with an appropriate characterization of the phenomena.

Of course, the history of diamagnetism extends back far beyond the episode we are concerned with, at least to Faraday. Interestingly, one finds a suggestive discussion of it in Maxwell’s classic Treatise on Electricity and Magnetism in terms of ‘molecular currents’ (Maxwell (1904), p. 418ff). Indeed, he notes that according to Weber’s theory of diamagnetism, “there exist in the molecules of diamagnetic substances certain channels round which an electric current can circulate without resistance. It is manifest that if we suppose these channels to traverse the molecule in every direction, this amounts to making the molecule a perfect conductor” (ibid., p. 421). Maxwell then proceeds to explore the consequences of a ‘mathematical conception’ of perfectly conducting bodies, bringing together Weber’s theory with Ampere’s theory of magnetism as due to electric currents. Thus we see that the background to the diamagnetic analogy takes us to Maxwell’s theory which also, not surprisingly, plays a crucial role in the construction of the London and London model. So, what we have is a rich theoretical set of background structures encompassing both classical and quantum analyses.

This analogy was explicitly noted in the crucial papers produced by the Londons in 1935. In his ‘solo’ paper, of 1935, Fritz London wrote:

It is rather seductive to consider the supra-current as a kind of diamagnetic current, an idea which has sometimes been uttered in the past, now more seductive than ever, since Meissner’s experiment seems to reveal to us the more elementary phenomenon to which one may hope to reduce the so enigmatical phenomenon of conductivity (1935, p. 26).

The analogy also features prominently in the 1935 joint paper:

In contrast to the customary conception that in a superconductor a current may persist without being maintained by an electric or magnetic field, the current is characterized as a kind of diamagnetic volume current, the existence of which is necessarily dependent upon the presence of a magnetic field (London and London, 1935, p. 88).

Furthermore, the nature of the analogy is that of an iconic model. Thus, London notes that “the magnetic behaviour of a superconductor resembles that of a very strongly diamagnetic metal” (London 1935, p. 26; our emphasis), and, as we shall see, in his 1937 article for Nature, he placed the analogy at the core of

2 Although the relevant background has been explored elsewhere, the details of this extension of the London model has not, to our knowledge, been discussed.
his analysis and explicitly obtained the central elements of the London–London model from a consideration of the behavior of a diamagnetic atom in a magnetic field (London 1937, p. 794). Some years later, he summarized the London–London model as a kind of generalization of the well-known relation between the diamagnetic current and the magnetic field within a diamagnetic atom (London 1950, p. 28). Below, we describe these developments and the increasingly significant role of the analogy in the characterization of the phenomena.

4. The London and London ‘schema’

Let us now consider the relations between the various models, Maxwell’s equations and the diamagnetic analogy. Note that our aim is not to offer a reformulation of the London model in terms of partial structures, but rather to track the relations between the several models drawing on the relevant aspects of them where appropriate.3

In the 1935 joint paper, London and London begin with the acceleration equation that defines the older model of superconductivity:

$$A\frac{dJ}{dt} = E$$

(1)

where $$A = m/nc^2$$, m is the mass of the electron, e the charge, n the number of electrons per cubic centimeter.

As they note, this expresses the effect of the electric component of the Lorentz force on freely movable electrons and when $$E = 0$$, stationary currents are possible. In our framework, this can be represented by the $$R_0$$, in the relevant model, namely those relations that are accepted, given the model. From (1) and applying one of Maxwell’s equations—namely curl $$E = -1/c \frac{dH}{dt}$$—they obtain:

$$\text{curl } A \frac{dJ}{dt} = -1/c \text{curl } H$$

(2)

Applying $$-1/c J = \text{curl } H$$, ignoring the displacement current and integrating, they obtain the inhomogeneous equation4:

$$\Delta \nabla^2 (H - H_0) = 0$$

(3)

Note, that only part of Maxwell’s theory is used to obtain this result, and various material features are ignored with only some relations carried over. So here we have the piecemeal application of ‘fragments’ of the theory. Furthermore, the analogy that was associated with this model was that of ferromagnetism, with the most stable state of the superconductor, below the critical temperature, corresponding to a permanent current, taken as analogous to the most stable state of the metal, below the Curie point, which is represented by a permanent magnetization (see, for example, London 1935, p. 25). As French and Ladyman noted, the theoretical conclusion reached by Bloch, which represented a huge stumbling block for developments at the time, namely that superconductivity should have been impossible, was based precisely on taking this analogy seriously. Just as a ferromagnet shows zero magnetization unless its domains are organized by some external field, so a superconductor, regarded as divided into analogous domains in each of which current would flow, would show zero overall current in the absence of a field (French and Ladyman, 1997, p. 375).

The inhomogeneous equation has a particular solution $$H = H_0$$, where $$H_0$$ is just a constant of integration, representing the original field. But this represents the field being ‘frozen in’ the superconductor, and is falsified by the Meissner experiment. London and London’s response is that the acceleration Eq. (1) gives “too general a description”, and “contains too many possibilities, as it gives nature more freedom than it wants” (London and London 1935, p. 73). As they say, since the Meissner effect implies that magnetic fields are not found in the superconducting phase, $$H_0$$ should be set to zero. Then comes the crucial passage:

If in reality $$H_0$$ is always confined to the value zero, then this means that $$\Delta \nabla^2 H = 0$$ is to be considered as a fundamental law and not to be treated as a particular integral of a differential equation in consequence of (1), (London and London, 1935, p. 73).

This equation:

$$\Delta \nabla^2 H = H$$

(4)

is the ‘homogenous’ equation and represents their abandonment of the acceleration equation and hence the old model. Note that this move to set $$H_0$$ to zero is driven by an experimental result. Accommodating this and other results was, of course, an absolutely central feature of London and London’s endeavor. Other phenomena referred to were the existence of permanent currents in a ring and the current obtained without an electric field in an open superconducting wire connected by normal conducting leads. As Fritz London emphasized in a further paper in 1937, “the consistent representation of these experiments was the basis of our theory” (1937, p. 835).

Thus the Meissner result is used as the basis on which the homogeneous equation can be regarded as a fundamental law. The issue then is what motivates so taking this equation? Here is where the diamagnetic analogy enters, as we shall now see.

Since curl $$H = 1/c J$$, from Maxwell’s equations again, the homogeneous equation above can be rewritten as

$$\text{curl } A = -1/c H$$

(5)

which they present as the ‘fundamental’ equation for superconductors. As London noted in 1937:

The magnetic field of these [superconducting] rings, having a curl, requires, according to Maxwell’s theory, the explicit introduction of the macroscopic current (London 1937, p. 796).

Alternatively, as he put it earlier, in his 1935 ‘solo’ paper, on this model superconductors are now characterized not by a particular value of the ‘permeability’ but by the above differential equation. This means that the superconductor can be regarded as “a single big diamagnetic atom” (1935, p. 27). In other words, what Eq. (5) gives us is the bridge or relationship to the theory of diamagnetism.

5. The role of analogy

Moving on to the analogy with diamagnetism, not all the features of the latter were drawn upon by the Londons at this stage. In particular, those aspects of the diamagnetic structure that relate to the microscopic understanding were not carried over (these can be captured by the $$R_0$$ components in our partial structures), not least because it would be inappropriate to do so in the construction of a ‘macroscopic’ models such as this. However, this further structure would prove to be of critical importance for the development of the ‘microscopical’ interpretation that Fritz London, in particular, went on to pursue.

 Crucially, then, since diamagnetism is a ‘microscopic’ (and ultimately, quantum) phenomenon, the analogy also functioned in a heuristic way for the construction of London’s attempt at delineating a ‘microscopic’ interpretation of superconductivity.

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3 One could in principle produce such a reformulation by representing the electro-magnetic field in terms of functions of space-time to real number triples. However, that would not serve our purposes here, since we are emphasizing the relations between the models and the importance of the diamagnetic analogy.

4 In the equation above, $$\nabla^2$$ is to be understood as curl of curl.
(French and Ladyman, 1997, p. 389). In this regard, the ‘neutral’ analogy, to use Hesse’s phrase, and as represented by the $R_k$ in our partial structures, was absolutely crucial as London explored the analogy in order to obtain his interpretation. We shall come to this aspect shortly.

Thus what we have in 1935 is the carrying over of just the relationship between the magnetic field and the current from the diamagnetic domain. If we carry our partial structures as follows:

\[
A = \langle D, R_k \rangle_{k \in K} \quad \text{and} \quad A' = \langle D', R'_k \rangle_{k \in K}
\]

then $D$ contains superconducting rings, $D'$ includes diamagnetic atoms, and the functions that the structures share, expressed by some relations $R_k$ and $R'_k$ respectively, are related by Eq. (5), which holds in both structures.

From Eq. (5), using again one of Maxwell’s equations, we can also recover Eq. (2), which, as noted above, was obtained on the basis of the old model. Thus, we get Eq. (2) from both the old and new models. Indeed London and London (1935, p. 74) represented the relationship between their model and its predecessor as follows (Fig. 1):

![Fig. 1. The old ferromagnetic-based model and the new diamagnetic-based model related.](Image)

London and London insist that Eq. (5) and Eq. (1) should be taken as possessing the same degree of generality (and thus Eq. (5) should be regarded as fundamental). However, as they note, Eq. (5) goes beyond Eq. (1) in one respect, since it accommodates the Meissner effect, but in another respect, it covers less given that we cannot derive Eq. (1) from Eq. (5). The relationship between the two models is mediated by Eq. (2), which, as they observe, can be derived from both of them. The significance of Eq. (2) for our discussion here is that it captures what was right about the old model and in effect represents the extent to which the old model was successful. What is then carried over from the old model to the new (together with its consequences) is the following statement (London and London, 1935, p. 74), weaker than Eq. (1):

\[
\text{curl } \frac{\partial A}{\partial t} = - E = \text{grad } \mu
\]

where $\mu/A$ gives the charge density (for further details, see French and Ladyman, 1997).

In going beyond Eq. (2), the acceleration equation “expresses a prejudice”, since it is not supported by experience and thus the heuristic moves made seem to fit what Heinz Post called ‘stripping’, where the old theory is stripped of its dispensable, not independently confirmed, superstructure (Post, 1971). What Eq. (2) effectively represents is the well-confirmed part of the old model which is recovered within, and thus inherited by, the new one. This stripping was guided by a combination of empirical considerations (namely, the Meissner effect) and the diamagnetic analogy, where the latter effectively determines the level at which the stripping stops.

Furthermore, we have two kinds of preservation in this situation: first of all, the relevant results and consequences represented by Eq. (2) are preserved; and secondly, we have structure preservation at the level of Maxwell’s equations. In both cases, the appropriate way of capturing these relations of preservation is via partial isomorphisms.

In effect what London and London are doing here is moving back and forth between the formalism and its interpretation, drawing on both theoretical elements from Maxwell’s equations and the diamagnetic analogy. Here Maxwell’s equations function as a mathematical device, or rule of inference, in order to get the inhomogeneous and homogeneous equations. To take the latter seriously, and elevate it to a fundamental law, a different understanding of superconductivity was needed, and here London and London drew on the diamagnetic analogy. The Meissner effect, as we noted above, is built into their model via $\mathbf{H}_0 = 0$. And this ends the characterization stage of the relevant phenomena. However, in order then to explain the phenomenon—that is, to account for why it emerges in the way it does—London realized that he needed to draw on some inner structure and go beyond the macroscopic interpretation. This is what he began to do in the period from 1935 to 1937, thereby moving to the explanatory stage.

### 6. Diamagnetism and the macro-micro move

As French and Ladyman (1997) noted, it was the diamagnetic analogy, that ‘set’ the program to obtain a microscopic interpretation. The analogy does not, of course, determine the precise form of the mechanism involved, but that there had to be some such program was so determined, given the quantum explanation of diamagnetism itself. Here we shall extend French and Ladyman’s account by tracking the developing role of the analogy as it drove a ‘reduction’ of the possible mechanism for explaining superconductivity.

This move to a microscopic interpretation first appears toward the end of the 1935 joint paper. There we find London and London noting the structural similarity between the fundamental equations of their model and “Gordon’s formulae for electric current and charge in his relativistic formulation of Schrödinger’s Theory” (London and London, 1935, p. 86). Following through on this they note that only a very weak (standard) form of diamagnetism would result, unless the electrons were taken to be coupled by some form of interaction. (It is this that has been taken as an early suggestion of the possibility of what came to be known as ‘Cooper pairs’.)

The structural relationship in this case can be represented by the appropriate partial isomorphism holding between the ‘macroscopic’ and ‘microscopic’ structures. Indeed, as we have emphasized, the microscopic program is at least partly structured by this relationship through the heuristic force of the analogy. What is crucial here is the fact that the same fundamental Eq. (5) holds for both domains. However, this is not sufficient to effect a complete structuring of the explanatory model to be delineated shortly, since it could not provide a mechanism to account for the electron coupling. In this case the structure is also partial, given that further theoretical developments were required to fill in the details of the mechanism (French and Ladyman, 1997, p. 390).

In London’s (1935) ‘solo’ paper, the relationship with the diamagnetic analogy is made explicit. After demonstrating the structural similarity, he writes: “incidentally, this mechanism is in no way an absolutely new one. It characterizes the whole superconductor as a single big diamagnetic atom” (London, 1935, p. 32). Of course, the superconductor is no such thing; so, again, we are talking of an analogy here (only some structure is, after all, transferred from the diamagnetic domain to that of the superconductor). In a sense, this paper represents an intermediate step in developing the program. London begins by simply stating the basic feature of the London–London model, in the form of Eq. (5).

After exploring certain consequences, he then notes that although the development of the theoretical foundation of this model had not yet been undertaken, “it is rather attractive to try to sketch the programme which seems to be set by our equations to a future microscopic analysis” (London, 1935, p. 31).

Acknowledging the need for some form of coupling mechanism, he then invites us to “consider an atom of the size of a ring” (1935, p. 32), and investigates the form of the current that would
be obtained, on quantum mechanical grounds, when such a ring has a given magnetic flux through its hollow part. He observes that the expression obtained has exactly the same form as the expression for superconducting current flow. As we just noted, he has already stated that the superconductor can be regarded as like one big diamagnet. Here the analogy becomes manifest as he concludes that “the persistent currents in a ring are therefore really to be conceived as a diamagnetic phenomenon which stabilises itself” (1935, p. 33).

In his 1937 paper, published in two parts, the program unfolds still further. Here we find London drawing on the quantum mechanical account of diamagnetism, right from the beginning, together with the analogy, to give the outlines of an explanation of superconductivity. Thus, after noting that the Meissner effect compels us to find a new model for superconductivity, he explicitly considers a diamagnetic atom and notes that the possibility of permanent currents in such a case is not covered by Bloch’s impossibility result, since such currents require the presence of an external field and the latter result covers only systems with no such field. He considers the behavior of such an atom in a magnetic field, and notes the following two essential properties it must possess (London, 1937, p. 794):

(a) Its lowest state is not degenerate and lies on a discontinuous spectrum.
(b) In a weak magnetic field, the wave function does not experience perturbations stronger than those proportional to the square of the field strength or higher. Thus, we have

$$\psi = \psi_0 + \hbar^2 \psi_1,$$

where $$\psi_0$$ is the wave function for $$\mathbf{h} = 0$$, and $$\mathbf{h}$$ is taken to be a weak magnetic field.

He then moves on to consider a superconductor and, crucially, assumes that in such a material there are electronic states that have the same properties (a) and (b). And here again the analogy explicitly manifests itself. He realizes that there is a kind of screening effect that prevents the diamagnetic current from dissipating due to lattice vibrations, because the magnetic field generates almost the same current in all the electron states. And this, he notes, is exactly the same mechanism by which interactions with nuclear vibrations in a diamagnetic atom are prevented from dissipating the diamagnetic currents. So here again we see further structure being imported from the diamagnetic domain to that of the superconductor.

This then licenses his taking as valid the same expression for a superconducting electron as for a diamagnetic atom. Taking into account all electrons in the superconductor, he obtains the relevant expression for the current density, and taking the curl of this, he obtains Eq. (5) of the London and London model once again, which he describes as the “fundamental macroscopic connexion” between the magnetic field and current density for a superconductor (1937, p. 795). As he goes on to note, assuming conditions (a) and (b) certainly amounts to a significant “reduction” (his word) of the mechanism which, he acknowledges, still remains to be explained by the theory of electrons. Of course, these conditions provide by no means a necessary basis for Eq. (5), and he admits that further developments may offer a still more reduced basis. As is well known, this work, and the diamagnetic analogy in particular, had a profound influence on Bardeen in his attempt to explain superconductivity from first principles (see, for example, Hoddeson, 2001). As Gavroglu notes, in the context of this attempt, the idea of coupled electrons that the Londons suggested came to be expressed in the concept of ‘Cooper pairs’ (Gavroglu, 1995, p. 209).

Thus, in 1937, the rest of the diamagnetic structure is brought over, so that we obtain something close to an isomorphism between the relevant structures. We say ‘close’ because, of course, a superconducting ring is not a diamagnetic atom, and thus we still have an analogy here. Initially this is part of the general background, and is used heuristically to promote the core equation of the new model to fundamental status. Here the partiality lies in the form of Eq. (5) and the characterization of superconductivity in terms of a differential equation: the rest of the theory of diamagnetism is effectively ignored at this stage. Indeed, it can be placed among the $$R_3$$ components of a partial structure, or what Hesse called the ‘neutral’ analogy. Through the structural similarity with the relativistic Schrödinger equation, this analogy then helps set the limits of the microscopic explanatory ‘reduction’. The partiality of the analogy as a heuristic device then becomes something stronger as elements from the neutral analogy, which we place among the $$R_3$$, are brought over into the positive analogy, or the family $$R_1$$. As we proceed from 1935 to 1937, the analogy becomes stronger and the partiality continues to decrease, as structure is brought over from the diamagnetic ‘domain’ to that of the superconductor. When the macroscopic equation of superconductivity is obtained on the basis of microscopic considerations (subject to the assumption of conditions (a) and (b), of course), the further structure is then effectively transferred over. We can think of the diamagnetic analogy as providing the channel for this importing of further structure which shapes the developing microscopic interpretation. By this stage, the analogy has helped London uncover the mechanism that he and his brother had only speculated about in 1935. Now it is providing a partial explanation of the Meissner result—partial because conditions (a) and (b) had not been established.

This case study clearly illustrates the two stages of the characterization-explanatory framework. As we saw, the Londons characterized superconductive phenomena via a combination of empirical and theoretical considerations, relying respectively on the Meissner effect and Maxwell’s equations. Fritz London then moved on to the explanatory stage by transferring not only some structure from diamagnetism, but also relevant microscopic components, that were invoked in order to obtain the fundamental equation of superconductivity. Throughout this process, as we saw, partiality of information was widespread.

It is useful to compare our account with van Fraassen’s influential discussion of theory construction. In particular, the significance and role of empirical factors, in the form of the Meissner experiment, suggests that what we have here is a form of what van Fraassen (1980) called “theory construction by other means”. He gives the example of Millikan’s oil drop experiment that accurately and unambiguously determined a unique value for the electric charge, where this value was theoretically an open question. In this case, on van Fraassen’s reconstruction, one has a theory that is already partly constructed and with gaps that need to be filled. Filling such gaps then takes theory construction one step further.

This feeds into a two-stage process of theory construction. The first stage involves a widening of the existing theoretical framework so as to accommodate the possibility of the new phenomena, rendering the theory empirically adequate. The second stage is a narrowing, to exclude many of these admitted possibilities, and regain empirical import and predictive power. However, the London case does not precisely match van Fraassen’s account. The move from the old model to the London–London one certainly did not involve a widening. On the contrary, as London and London noted, the inhomogeneous equation gives “too general a description”, and in moving from Eq. (1) to Eq. (5) in order to accommodate the Meissner effect, we reduce the options available. The difference, of course, is that we do not have an existing theoretical framework that is widened. Rather we are concerned with the replacement of one macroscopic characterization of a phenomenon with another. On the
other hand, the move to a microscopic interpretation, at the explanatory stage, could be understood as a partial narrowing in the sense that it excludes many possibilities and sets out a program for generating a quantum model with full predictive power and empirical import.

Thus we see that the way in which the Londons obtained the core equation of their model is not straightforward. Maxwell's theory in effect yields too many possibilities, as the Londons put it, and the Meissner effect has to be called upon to select one of these, with the help of the diamagnetic analogy. But, of course, without Maxwell's equations the Londons could not have obtained their core equation at all. Obviously we do not have a simple derivation in the sense that Eq. (5) can be obtained from Maxwell's equations alone. If that were the case, the Meissner effect would not have the significance that it has! But it is not so unusual in science to find equations and theorems obtained from very basic or general theories with the conjunction of other principles, empirical statements and so forth.

There are a number of features of the above account that are worth developing further.

7. Further developments

7.1. The role of analogy

In one sense we can say that a superconductor can be conceived of as a kind of diamagnet, but in another sense, as London himself makes clear, it is not—it is not really a giant diamagnetic atom, for example. The crucial point is the characterization of superconducting current as a kind of diamagnetic current. There are two ways of understanding 'kind' in this context. In one sense, 'kind' is indicative of the analogical nature of the relation with the diamagnetic, and in this sense, 'kind' signifies that only aspects of the diamagnetic conception are carried over. In another sense, 'kind' suggests that a superconductor is a type of diamagnet. Rather than simply an analogy, we have here a classification, where a superconductor is identified as belonging to a particular type of objects, namely, diamagnets. Which sense is being used here?

Both senses can be found in this case. But it is important to realize that even when the classification/identification sense is at stake, there is still an analogy in place. In 1935, as we saw above, only some aspects of the diamagnetic conception are transferred to the superconducting view—these aspects had to do with the relationship between the magnetic field and the current density. However, at this point, the particular microscopic details associated with diamagnetism were not carried over. And even when further microscopic structure was eventually transferred—that is, in 1937—still not every structural component was brought to the superconducting conception. After all, as just noted, a superconductor is not literally a huge diamagnetic atom. That is, even when a superconductor was identified as a kind of diamagnet, only some of the relevant relations were carried over, others were not. And it is this partial carrying over that can be captured by suitable partial morphisms in the context of the partial structures account, and which underlie the analogical nature of the relation between a superconductor and a diamagnet.

7.2. Ad hoc-ness and truth

Our summary above reveals the empirical underpinning of the London and London model but it would be a mistake to read from this that the model was ad hoc in representing what it was supposed to explain. First of all, this would conflict with Fritz London's own two-stage account of theory construction where the first established what he called a 'macroscopic' interpretation that not only had to be consistent with the relevant phenomena but also had to embrace theoretical elements, methods and aims. (As we have argued, this move corresponds to our characterization stage.) The London and London model was not intended to explain the Meissner phenomenon but rather, as we have emphasized, was proposed as a new characterization of it, on the basis of which one could then seek an appropriate explanation in terms of what London called the 'microscopic' interpretation.

Of course, if it were ad hoc, this would compromise the ability of the model to transmit its empirical success to the theory used in its construction. In this case, the accommodation of novel phenomena, such as the Meissner effect, would speak only to the instrumental reliability of the model, not its truth, because for the latter the phenomena must feature as the result of a novel prediction, rather than accommodated within the model in an ad hoc fashion. However, neither we, nor London, would take the Meissner effect as speaking to the truth of the model, not because the model is ad hoc, but because it is only the first stage in the development of a viable theory of superconductivity and was never intended, as we said, to explain the phenomena. Nor would we, or London we suspect, be troubled by the compromised ability of the model to transmit its empirical success to Maxwell's theory. This was never a significant issue. The aim was to construct a model that appropriately represented the phenomena, using both Maxwell's equations and the diamagnetic analogy as we have seen. (We shall return to the aim of the characterization stage below.)

7.3. Piecemeal borrowing

Suárez and Cartwright (2008) have usefully suggested that what we see in the construction of the initial London and London model (corresponding to our characterization stage) is "a kind of piecemeal borrowing from an old model of superconductivity to the new London model; borrowing that takes some assumptions but leaves others behind" (2008, p. 63). Our account above is intended to indicate the way that borrowing was informed by various factors, empirical as well as theoretical. Of course, the role of the latter suggests that the construction of the London and London model was not detached from theoretical considerations. Interestingly, this is borne out by the principal participants' own understanding of how the model was constructed. Fritz London was, of course, an unusually reflective physicist, with a rich philosophical background, and it is worth taking seriously his own view of what he and his brother were doing. In particular, in his classic text *Superfluids* (London, 1950) London makes it clear that the model "actually goes considerably beyond what is given by measurements on matter in bulk" (London, 1950, p. 30; his emphasis; see French and Ladyman, 1997 for further discussion). On being awarded the Lorentz Medal in 1953, London reiterated this point and made it clear that the equations "go beyond what is directly given by the phenomena" (Gavooglu 1995, p. 144; see also pp. 247–252). And they go beyond precisely through drawing

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5 Truth is not a significant issue in this context either for us—who consider the appropriate notion to be that of quasi-truth (see French and Ladyman, 1997, pp. 390–391, and Bueno, 1997)—and who interpret this from a realist and empiricist stance, respectively—or London, who had a phenomenologist's view (in the Husserlian sense) of such matters.

6 This is not to re-introduce intentions into the discussion, or at least not in a way that requires representation in one's philosophy of science. The point is, if the principal scientist involved in the model construction—someone who was, moreover, philosophically very reflective—insists that this model was not phenomenological, contrary to a particular stance in the philosophy of science, then that insistence needs to be explained away, at the very least.
on the diamagnetic analogy to help shape the next, explanatory stage of theorizing.

He also made it clear that the aim of his work with his brother was to develop what he called a ‘macroscopic’ description or interpretation of superconductivity that would function as the necessary characterization of the phenomenon prior to developing a new theoretical or, as he called it, microscopical, explanation, corresponding to our characterization and explanatory stages, of course. The ‘sketch’ of London’s explanation, although deficient in not offering all the details of the mechanism involved, did at least reduce the range of possible such mechanisms, as we have discussed.

7.4. Autonomy

There has been considerable discussion in recent years regarding the supposed ‘autonomy’ of scientific models from the relevant theory. It is worth considering further the sense in which the London and London model might be regarded as ‘autonomous’ (Suárez and Cartwright, 2008, p. 66). In its ordinary usage ‘autonomy’ means something like self-government. Of course, models cannot be taken to govern themselves because theories are necessary to provide the theoretical concepts and laws used in them. It is not as if what magnetic flux or resistance does in a model is independent of how the theory of electromagnetism characterises those things. In any case, we will consider four possible senses in which models might be said to be autonomous and shall determine which holds in the case of the London and London model (these considerations build on but go beyond those found in Bueno, French and Ladyman, 2002; da Costa and French, 2003).

Autonomy 1: Model $M$ is autonomous from scientific theory $T$ if and only if it is not derived, obtained from or otherwise related to $T$ by any acceptable move, where $T$ is any theory available.

The term ‘obtained’ here is ambiguous between ‘actually obtained’, in the sense of constructed at the time the model was initially put forward, and ‘subsequently obtained’, that is, constructed at a later time. In either case, this is a very strong sense of autonomy and we know of no cases where a model is autonomous from theory in this sense; certainly the London and London model was not.

We can then introduce restricted senses of autonomy by taking ‘obtained’ to mean at the time of model construction and/or taking $T$ to be some more fundamental theory that is known to be relevant to the context in which the model is proposed.

Autonomy 2a: $M$ is autonomous from $T$ if and only if at the time it was constructed, it could not be obtained from or related to $T$, where $T$ is any theory available.

Autonomy 2b: $M$ is autonomous from $T$ if and only if at the time it was constructed, it could not be obtained from or related to $T$, where $T$ is some relevant ‘high-level’ or more fundamental theory.

Autonomy 2a might be satisfied in the case of a model that is constructed purely ‘from the ground up’, although again we can think of no such cases, and yet again this was not the case for the London and London model. Autonomy 2b, on the other hand, does seem to match some cases in the history of science, such as the liquid drop model of the nucleus (see da Costa and French, 2003). Here the relevant high-level theory might be understood as a detailed quantum mechanically informed theory of the nucleus and that simply was not available at the time the model was constructed. Indeed, the liquid drop model might be seen as another case of a ‘characterization’ model, but where the role of analogy (in the form of the comparison with a liquid drop) was even more pronounced. In this sense, the London and London model might be seen as autonomous, with the subsequent development by London reducing, but not completely eliminating, that autonomy by indicating how the model could be obtained from a more fundamental quantum mechanical description. However, this form of autonomy seems to us unproblematic, not least because it is temporary (Bueno, French and Ladyman, 2002): as theoretical developments proceed, the autonomy is reduced.

Autonomy 3: Model $M$ is autonomous from theory $T$ if and only if it is not obtained from $T$ by an appropriate set of de-idealizations applied to $T$.

Again, we can obtain two variants depending on whether we take the de-idealizing move to be applied as part of the construction of the model (Suárez and Cartwright, 2008, p. 68), or subsequently, as it is realized that the already constructed model can be related to some theory by such a move.

In either case, this is obviously a more restricted sense of autonomy, and certainly the London and London model was not obtained by de-idealizing moves from high-level theory. But, of course, models in general and the London and London model in particular are typically tied to theory through other kinds of moves. And in this sense, they are not autonomous from theory. Furthermore, given that the London and London model was characterized in the way we have indicated, one should not expect it to be obtained “by improvements legitimated by independently acceptable descriptions of the phenomena” (Suárez and Cartwright, 2008, p. 68), since it was itself intended to provide just such an acceptable description.

Autonomy 4: $M$ is autonomous from $T$ in the sense that it acts as the locus of epistemic activities.

This is the view of autonomy that Morrison holds when she states that “models function in a way that is partially independent of theory” (1999, p. 43; her emphasis). As she notes, it is the appropriate autonomy in model construction that gives rise to such functional autonomy, but even when theory does play a significant role in model construction it is still possible to have the kind of functional independence that renders the model an autonomous agent in knowledge production (Morrison 1999, p. 43).

This is a very general sense of autonomy, and again (leaving aside issues as to whether models can be regarded as ‘agents’) is uncontentious. Indeed, it is surely a matter of common agreement that the roles of models in science may include acting as the locus for knowledge claims, providing the basis for further developments and so on. In this sense, the London and London model, and the explanatory structure it led to, can certainly be regarded as functionally autonomous.

Thus, we conclude that the London and London model was autonomous from theory in senses 2b, 3 and 4. However, these can be regarded as uncontentious, either because the autonomy is temporary, or because the model was never intended to be obtained via de-idealization, or because the relevant sense of autonomy relates to the way in which models become the focus of scientific practice. The London and London model was not autonomous in the sense of either 1 or 2a, but these strike us as problematic anyway.
7.5. Representing the relationships between models

The framework we have presented here is broadly unitary in taking there to be fundamentally one kind of relation (e.g., partial isomorphism) that can be used to represent the relationships that hold between all models. Thus a universal account of modeling can be obtained in these terms, that is, in terms of partial structures and other mathematical structures to be captured via partial homomorphisms (Bueno, French and Ladyman, 2002). There may be examples where it can be argued something other than partial iso- or homo-morphisms are required, and in that case we would hope that the partial structures program could be further expanded to accommodate these further morphisms. Furthermore, there are of course other versions of the semantic approach on the market, as it were, and it may be that it can be argued that one of these is better at capturing certain aspects of scientific practice. In that case, we would be prepared to adopt a more pluralist position, along the lines we have already adopted with regard to category theory (see da Costa and French, 2003, p. 26). This may well be a more appropriate framework for representing certain structuralist commitments, but we believe our set-theoretic account properly accommodates the kinds of theory–theory and theory–data relationships that the philosopher of science is interested in (see French forthcoming).

An alternative suggestion is that for the given models, a set of relations can be found that will hold between them, but this set will be different for different cases (Suárez and Cartwright, 2008, p. 78). Thus one may adopt a Suppesian hierarchy of levels of models à la Bueno (1997), but insist that each layer in the hierarchy may involve a change of elements in the relevant domain or a change in the set of relations. The move from one level to the next is not grounded upon any features of the structures themselves or on the relations that hold between them but upon what Kaiser called “inference tickets” (Kaiser, 1991). This is an interesting suggestion and there is nothing in the partial structures approach per se that is inherently incompatible with the role of ‘inference tickets’ (cf. da Costa and French, 2003, Chapter 4). And this is because such tickets are not themselves primitive or ungrounded, but ride on the back of the relevant relationships between the structures, relationships that can then be captured by partial isomorphisms, homomorphisms, or other morphisms. However, the following question then arises: if the move from one structure to the other is not grounded upon any features of the structures themselves, then what licenses the issuing of the relevant inference ticket? Suppose the given structures did not possess any of the relevant features that would license a move from one to the other; then obviously no such move could be justified (consider, for example, a structure plucked from loop quantum gravity and another taken from molecular biology). If they do possess the relevant features, then appropriate relationships can be established between them and those can in turn be represented, within the set-theoretic framework of the semantic approach, by the kinds of relations we have been discussing here.

What is being suggested here seems open to the same kind of concern that can be raised with regard to the ‘methodological minimal structuralism’ of Brading and Landry (2007). There the idea is that we should not impose a prior set-theoretic framework on scientific practice, but should take our lead from science itself in determining the kinds of ‘shared structures’ that obtain. The worry here (see, for example, French forthcoming) is that this yields an approach to the analysis of scientific practice whereby we simply ‘read off’ the relevant relationships as they are presented at the level of this practice itself. (Here we might recall the Natural Ontological Attitude’s insistence that we ‘let science speak for itself.’) The concern is that such reading off may result in, not a rich, vibrant tapestry, but a confused, inchoate and tangled web from which little sense can be made (or, in the case of the NOA, a cacophony of voices). We take it that what we are doing in the philosophy of science is selecting those features of scientific practice that we deem of most interest, relative to our philosophical aims, of course, and describing or representing them in the most appropriate way to meet those aims.

Fundamentally, then, there is a need to be clear about the nature of the game here: it is not the case that we have, in some sense, ‘raw’ structures and we then seek to establish set-theoretic relations between them. Rather it is that we—philosophers of science, that is—are presented with the theories and models that feature in scientific practice and we then characterize, or, if one prefers, represent, these elements in such a way as to illuminate those features that we, as philosophers of science, are interested in. Of course different philosophers of science may be interested in different features of science and perhaps some form of pluralism might be adopted. Nevertheless, we think that for those features that we take to be central to the philosophy of science—namely those that have to do with the inter-relationships between theories, between theories and data models, between theories and mathematical structures, and so on—the partial structures approach not only offers a unitary framework, in the sense of capturing relevant aspects of those inter-relationships, but it also offers the best such framework, since its inherent partiality allows it to accommodate the openness, multifacetedness, and complexity of the scientific enterprise.

8. Conclusion

What we have in the London and London case is an interesting study of the combination of empirical and heuristic factors contributing to the construction of a new characterization of a phenomenon in terms of what was called a ‘macroscopic’ model. This provides a useful example of what we have called the characterization stage of theorizing. These same heuristic factors—in the form of the diamagnetic analogy—then helped shape the further theoretical developments as the relevant structure was imported over from the diamagnetic domain to underpin the beginnings of a theoretical explanation of the phenomenon. Here we move to the explanatory stage, which in this case remained significantly open-ended until further theoretical work was undertaken. Thus these developments were multiply partial, in the sense of both relying on the transfer of ‘pieces’ of structure (in particular, from the domain of diamagnetic phenomena) and in being open-ended. We claim that such developments can be accommodated within the partial structures account, appropriately understood, and that this case study indicates how van Fraassen’s two-stage approach is too simplistic as it stands. In the end, in understanding such episodes, due attention must be paid to both the impact of empirical findings and the relevant heuristic moves.

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