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Why Inconsistency Is Not Hell

Making Room for Inconsistency in Science

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1. INTRODUCTION

On most accounts of belief change, inconsistent belief systems are an "epistemic hell" to be avoided at all costs (see, e.g., Gärdenfors 1988, p. 51). From a normative point of view, we can perhaps understand why this is the case. The underlying logic of most theories of belief change is classical, and classical logic is explosive, that is, everything follows from a contradiction. And a belief system from which everything follows should definitely be avoided. It is certainly of not much use if one wants to determine what one should believe and what one should do.

In a number of works, Issac Levi challenged this way of approaching the issue. On his view, there are contexts in which inconsistent belief systems are bound to happen. This is the case, for example, of observations. According to Levi, in some contexts it is legitimate to add a doxastic proposition to a belief system with which it is inconsistent: "Making observations and coming to fully believe propositions incompatible with one's initial convictions is a case in point" (Levi 1991, p. 68). The idea is that we may inadvertently tumble into inconsistency as the result of "deploying a reliable program for routine expansion" (ibid., p. 110), that is, as the result of adding a new belief to our belief system. In other words, descriptively at least, inconsistent belief systems simply happen, and this fact needs to be accommodated.

The trouble, however, is that despite acknowledging that expansion into inconsistency may sometimes be legitimate, Levi immediately adds that "it is always urgent to contract from an inconsistent state of full belief. The contraction will remove either A, \( \sim A \), or both" (ibid., p. 68). This urgency clearly derives from the trivialization (in classical logic) of the belief system resulting from its inconsistency. And for this very reason, Levi insists that an inconsistent belief system is unacceptable, since it "fails as a standard for serious possibility for the purpose of subsequent inquiry and for practical deliberation" (ibid., pp. 76–7; see also Levi 1996).

Levi has certainly gone a long way toward devising a broader, more encompassing approach to belief change with regard to inconsistency. And he can only be commended for that. In this chapter, I argue that we can, and should, go further. It is not only that descriptively we need to make room for inconsistent belief systems -- they are indeed a striking fact of our epistemic life. But also normatively to maximize informativeness or, at least, to minimize information loss, we often need to entertain and pursue inconsistent belief systems. To pursue inconsistent systems is a useful device for a number of reasons: (1) This is often the only way to explore inconsistent information without arbitrarily rejecting precious data. (2) Pursuing inconsistent systems is sometimes the only way to obtain new information (particularly information that conflicts with deeply entrenched theories). As a result, (3) pursuing inconsistent belief systems allows us to make better informed decisions regarding which bits of information to accept or reject in the end.

After motivating and illustrating each of these three benefits of inconsistent belief systems, with examples from the foundations of mathematics and the empirical sciences, I argue that Levi's own approach to belief change could become stronger by making room for inconsistency, without giving up the crucial features of his pragmatism. The crucial move is to change the underlying logic to a paraconsistent one, which, in consistent contexts, yields exactly the same results as the classical approach (see, e.g., da Costa and Bueno 1998). Once there is room for inconsistency, there is also room for informativeness in inconsistent belief systems. As a result, the emerging account accommodates both descriptively and normatively the

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1 See, in particular, Levi's recent response to Olsson's searching critique of his account of contraction from inconsistent belief states (Levi 2003 and Olsson 2003). If inconsistency is epistemic hell, Olsson asks how could it ever be rational to enter such a state, and how could one regain consistency? Levi resists the critique by noting that one could devise a routine "prior to inadvertent expansion into inconsistency when the deliberating agent embraces a consistent point of view" (Levi 2003, p. 141). This strategy, however, doesn't succeed in making room for inconsistency, given that it ultimately shifts the burden to the reliability of the presumed consistent belief systems prior to the expansion into inconsistency. An alternative approach is in order.
significant role that inconsistency plays in inquiry, without yielding an epistemic hell.  

2. BENEFITS AND COSTS OF INCONSISTENT BELIEF SYSTEMS

There are many reasons why we should take seriously inconsistent belief systems — and many benefits emerge from doing that, which I examine below. But there are also important reasons why we need to be extremely careful when dealing with such systems. So, before examining the benefits of inconsistent systems, it’s important to be clear about the alleged difficulties — and hence the costs — posed by inconsistency, and determine whether, and to what extent, they are reasonable.

2.1. Costs of Inconsistent Belief Systems

Three major costs are typically associated with inconsistency:

(i) Triviality. As we saw above, a common charge against inconsistent systems is that they are trivial (given classical logic); that is, everything follows from them. So, inconsistent systems (whose underlying logic is classical) are useless as epistemic guides and as the basis for practical deliberation (particularly, given the goal of maximizing true beliefs and minimizing false ones). This is, of course, a major complaint, and ultimately it’s what has given inconsistency such a bad name.

(ii) Unreliability. A further reason why inconsistent systems are not considered epistemically significant emerges from the fact that the world itself is not taken to be “inconsistent” — in the sense that there cannot be true inconsistent descriptions of the world. Thus, if such descriptions cannot be true — or, at least, not completely true — there are no grounds to think that inconsistent theories are even reliable. After all, if no inconsistent theory of the world can be true, no inconsistent theory is true, even in the long run.

Furthermore, it’s also unclear how we could approach true, consistent theories via inconsistent ones. After all, if inconsistent theories are trivial (in classical logic), there’s no guidance as to which parts of an inconsistent theory should be preserved and developed further, and which should be rejected. Nothing in such a theory would decide that. Note that the

situation here is substantially different from the familiar cases raised by Duhem and Quine. Their point is that logic cannot decide which part of an empirically false theory should be rejected and which part should be further developed. But in the case of a consistent theory, one can in principle still draw nontrivial consequences from the “package” of the theory under test plus auxiliary assumptions, additional theories regarding the measuring devices, and so on. By exploring additional consequences of the overall “package,” one can, at least in principle, try to determine what’s the best way to proceed. However, the case of an inconsistent theory (whose underlying logic is classical) is importantly different. Given the triviality of the theory (in classical logic), there’s no way of exploring further consequences from the theory in any informative way: Everything follows from the theory. Under these circumstances, inconsistent theories are not of much use.

Moreover, if we take the aim of inquiry to be the production of true, maximally consistent belief systems (Levi 1991), then inconsistent belief systems are not an obvious starting point. In turn, if we start from maximally consistent belief systems, we may end up developing a true maximally consistent system in the long run. Given their lack of reliability, inconsistent systems are simply not the way to go.

(iii) Lack of informative. The charge of lack of informativeness of inconsistent belief systems is connected with the triviality complaint. If everything follows from an inconsistent system (in classical logic), such a system would be completely uninformative. We would be unable to use such a system to distinguish between bits of information that we have reason to believe — given the available evidence — and those that we haven’t. After all, in classical logic, an inconsistent system would seem to give us equal reason to believe in everything — definitely not a welcoming result!

But do we have good reasons to accept these complaints? As will become clear, I don’t think so. In fact, as long as we adopt the right framework to conceptualize and accommodate inconsistency, the alleged costs of inconsistent belief systems just discussed turn out to be significant benefits.

2.2. Benefits of Inconsistent Belief Systems

If we take seriously the examination of inconsistent belief systems, interesting benefits emerge. But note that to take such systems seriously, it’s not required to take them as true. There are many possible attitudes one can have toward a belief system besides truth. A system can be accepted as empirically adequate only, as quasi-true, or as warrantedly assertible. We can simply pursue the

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2 A note about the terminology: In this chapter, I use “inconsistency” and “contradiction” more or less interchangeably. Moreover, in the discussion of belief systems and belief change below, I also move more or less freely from talk of beliefs to talk of propositions that express such beliefs. Nothing of substance for the present discussion hangs on this.
inconsistent theories \textit{could} be true, and this is done independent of Priest's more radical interpretation that insists on the existence of true contradictions. Rather, as part of an overall empiricist approach to inquiry, if scientific theories need not be true to be good, in particular, inconsistent scientific theories need not be true either. Nothing in scientific practice \textit{requires} a commitment to the truth of the theories in question.\footnote{According to Jody Azzouni, the commitment to the truth of a scientific theory is indispensable, given that often we need to endorse a theory that, for a variety of reasons, cannot be explicitly stated (Azzouni 2004). Even if we grant him the point, it doesn't follow that the resulting notion of truth is substantive in any way (as Azzouni himself acknowledges). All we need in this case is a deflationary account of truth, which is essentially a logical device that allows us to assent to sentences in blind contexts (i.e., contexts in which the content of the relevant sentences has not been specified). But the commitments that a deflationary notion of truth brings (if any) need not be problematic at all (see Azzouni 2004).} And hence, nothing in such a practice demands a commitment to the truth of inconsistent theories.

Instead of simply outright banning inconsistent theories, we could use them to generate better, consistent successors, by exploring, in a paraconsistent setting, the consequences that inconsistent theories have. Only after such an exploration can we identify the features of the theories in question that we may have good reason to preserve. In this sense, properly explored, inconsistent theories may be reliable or, at least, could be used to generate better theories in the long run.

Note that throughout this process, the aim of inquiry can still be truth (or perhaps quasi-truth). But there's no reason to assume that a maximally consistent belief system is a better guide to truth than a maximally nontrivial belief system (see, e.g., da Costa 1997). In fact, once the distinction between inconsistency and triviality is drawn in a paraconsistent setting, a maximally nontrivial system (even though inconsistent) may be a better guide for the generation of a true maximally consistent system than a maximally consistent system alone. After all, with more information to explore in an inconsistent setting, better theories could be formulated.

(iii) \textit{Informativeness.} Once the triviality charge is accommodated, the complaint regarding informativeness can also be dealt with. Inconsistent belief systems need not be uninformative. After all, given that in a paraconsistent setting not every sentence follows from a contradiction, an inconsistent belief system won't provide us with (equal) reason to believe in everything. Some — but not all — sentences will be entailed by other sentences
in the system. But, to insist, there will always be sentences that are not so entailed. In this way, in a paraconsistent setting, the support given to the various beliefs of an inconsistent belief system is not uniform. There’s genuine information to be explored, and just as with a consistent system, some sentences are better supported and more entrenched than others. Hence, in a paraconsistent setting, typically even inconsistent belief systems can be informative.5

There are three additional benefits related to the informativeness of inconsistent belief systems that are worth noting:

(a) In a classical setting (one in which the underlying logic is classical), faced with an inconsistency, one is committed to reject more or less a priori some data — perhaps even valuable data.6 Exploring, in a nontrivial way, inconsistent belief systems provides an alternative way to investigate inconsistent information without arbitrarily rejecting available information. This provides a better mechanism to revise the original inconsistent system. After all, in a paraconsistent setting, it’s possible to make inferences nontrivially in the presence of inconsistencies. As a result, we would be in a better position to devise improved, consistent formulations of the original inconsistent system, without losing significant information along the way.

(b) Pursuing inconsistent systems is sometimes the only way to obtain new information, particularly information that conflicts with deeply entrenched theories (see Lakatos 1978 and Feyerabend 1988). For example, when Bohr developed his atomic model, he articulated an inconsistent proposal, given the accepted theories at the time. On the conception of the atom he adopted, an atom was thought of as a minuscule planetary system, with electrons orbiting round a positive nucleus. But, according to electromagnetism, an atom of that sort would be radically unstable and would almost immediately collapse.

Bohr essentially ignored the inconsistency, introducing by fiat a postulate to the effect that

[energy radiation [within the atom] is not emitted (or absorbed) in the continuous way assumed in the ordinary electrodynamics, but only during the passing of the systems between different “stationary” states.]

(Bohr 1913, p. 874)

Despite the inconsistency, Bohr’s model was not trivial. Bohr certainly didn’t derive everything from his model, but managed to obtain the correct predictions — for example, regarding the wavelengths of hydrogen’s line emission spectrum, among other items. Developing the model at the time was the only way to obtain such results, despite the inconsistency with accepted theories.7

A similar phenomenon is also found in nonempirical sciences, such as mathematics. For instance, as is well known, Frege developed a logicist reconstruction of arithmetic, but, inadvertently, in an inconsistent setting (Frege 1950, 1962). Significantly enough, though, Frege didn’t derive everything from his inconsistent principles, but managed to show how arithmetic could be reconstructed from second-order logic and some definitions. Moreover, at the time, Frege’s reconstruction was the only way of obtaining the logistic result. In fact, Russell’s theory of types hadn’t been developed yet, and even if it had been, it’s unclear whether that theory meets the logicist requirements introduced by Frege.

Recent attempts to save Frege from contradiction — for example, by jetisoning the inconsistent Basic Law V and taking Hume’s Principle as basic8 — have the benefit of restoring the consistency of Frege’s approach (see Wright 1983; Boolos 1998; and Hale and Wright 2001). But the resulting proposal has the cost of introducing a system that is not as obviously logicist as Frege’s. After all, it is at least a contentious issue whether Hume’s Principle is analytic, and so whether it could be legitimately considered an adequate basic principle for a logicist reconstruction of arithmetic.8

5 Of course, this will depend on the paraconsistent logic and the contradiction in question. After all, if only one sentence is not entailed by the contradiction, the logic is paraconsistent, but, in such a case, the resulting theory is still uninformative. However, the paraconsistent logics I have in mind here, such as da Costa’s C-logics (da Costa 1974), are more discriminative with regard to explosion than that.

6 It might be argued that, in the presence of an inconsistency, the procedure of rejecting some information is perfectly justified. Given that, assuming classical logic, no inconsistent belief system can be true, at least some information in that system must be false, and hence part of the system must be rejected — or so the argument goes. The trouble, however, is to determine which bits of information to reject, in a way that both preserves as much information as possible and doesn’t rule out bits of information arbitrarily. Much work in belief revision attempts to model this process. But being able to reason with inconsistent information provides an alternative pathway for that — including alternative strategies to obtain new, consistent versions of the original inconsistent system.

7 Lakatos has a fascinating, although somewhat idiosyncratic, discussion of this episode (see Lakatos 1978, pp. 55–68).

8 Roughly speaking, Basic Law V states that every concept has an extension (of the objects that fall under that concept). The only essential use of Basic Law V made by Frege was to obtain Hume’s Principle. According to that principle, two concepts are equinumerous if and only if there is a one-to-one correspondence between them. And using the latter principle, plus second-order logic and some definitions, Frege managed to provide a logicist reconstruction of arithmetic (Frege 1950, 1962). So, the idea was to reject the inconsistent Basic Law V, adopt Hume’s Principle as the basic principle for the logicist reconstruction of arithmetic, and carry out Frege’s approach from there (see Wright 1983; Boolos 1998; and Hale and Wright 2001).
Thus, as long as the appropriate logic is in place, there's no reason to think that inconsistent belief systems are trivial, unreliable, and uninformative. In the end, there are clear benefits associated with such systems.

3. MAKING ROOM FOR INCONSISTENCY: PARTIAL TRUTH AND PARACONSISTENCY

Having motivated some of the benefits of inconsistent belief systems, how can such benefits be achieved? What we need is a framework that (1) allows us to represent formally the idea that belief systems (in particular, scientific theories) need not be true to be good, and (2) accommodates inconsistent belief systems (in the sense that the latter are not trivialized in the presence of contradictions). To achieve (1), the notion of partial (or quasi-) truth is introduced. To achieve (2), an underlying paraconsistent logic is, of course, needed. Interestingly enough, given that the logic of quasi-truth is itself paraconsistent (see da Costa, Bueno, and French 1998), it's possible to develop a unified framework—the partial structures approach—that accommodates both (1) and (2). I briefly sketch this framework below.

The partial structures approach relies on three main notions: partial relation, partial structure, and quasi-truth (see, e.g., da Costa and French 1989, 1990, and 2003). One of the main motivations for introducing this proposal comes from the need for supplying a formal framework in which the openness and incompleteness of information dealt with in scientific practice can be accommodated (see da Costa and French 2003). This is accomplished by two moves. First, the usual notion of structure is extended. In order to model the partialness of information we have about a certain domain, the notion of a partial structure is introduced. Second, the Tarskian characterization of the concept of truth for partial contexts is put forward, which leads to the corresponding concept of quasi-truth (or partial truth).

To introduce a partial structure, the first step is to formulate an appropriate notion of partial relation. When investigating a certain domain of knowledge $\Delta$, we formulate a conceptual framework that helps us in systematizing and organizing the information we obtain about $\Delta$. This domain is tentatively represented by a set $D$ of objects and is studied by the examination of the relations holding among $D$'s elements. However, we often face the situation in which, given a certain relation $R$ defined over $D$, we do not know whether all the objects of $D$ (or n-tuples thereof) are related by $R$. This is part of the

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9 Further developments and applications of the partial structures approach can also be found, e.g., in Bueno 1997, 1999, and 2000 and da Costa et al. 1998.
incompleteness of our information about $\Delta$ and is formally accommodated by the concept of partial relation. More formally, let $D$ be a nonempty set; an $n$-place partial relation $R$ over $D$ is a triple $(R_1, R_2, R_3)$, where $R_1$, $R_2$, and $R_3$ are mutually disjoint sets, with $R_1 \cup R_2 \cup R_3 = D^*$, and such that $R_1$ is the set of $n$-tuples that (we know that) belong to $R$, $R_2$ is the set of $n$-tuples that (we know that) do not belong to $R$, and $R_3$ is the set of $n$-tuples about which we do not know whether they belong or not to $R$. (Note that if $R_3$ is empty, $R$ is a usual $n$-place relation that can be identified with $R_1$.)

However, in order to represent the information about the domain under consideration, we need a notion of structure. The following characterization, spelled out in terms of partial relations and based on the standard concept of structure, is meant to supply a notion that is broad enough to accommodate the partiality usually found in scientific practice. Partial relations do the main work, of course. A partial structure $S$ is an ordered pair $(D, (R_i)_{i \in I})$, where $D$ is a nonempty set, and $(R_i)_{i \in I}$ is a family of partial relations defined over $D$.

Two of the three basic notions of the partial structures approach have now been defined. To spell out the last and crucial one – quasi-truth – an auxiliary notion is required. The idea is to use, in the characterization of quasi-truth, the resources supplied by Tarski’s definition of truth. However, since the latter is defined only for full structures, we have to introduce an intermediary notion of structure to “link” full to partial structures. And this is the first role of those structures that extend a partial structure $A$ into a full, total structure (which are called $A$-normal structures). Their second role is purely model-theoretic, namely, to put forward an interpretation of a given language and, in terms of that interpretation, to characterize basic semantic notions. $A$-normal structures are defined as follows: Let $A = (D, (R_i)_{i \in I})$ be a partial structure. We say that the structure $B = (D', (R'_i)_{i \in I})$ is an $A$-normal structure if (1) $D = D'$, (2) every constant of the language in question is interpreted by the same object both in $A$ and in $B$, and (3) $R'_i$ extends the corresponding relation $R_i$ (in the sense that each $R'_i$, supposed of arity $n$, is defined for all $n$-tuples of elements of $D'$). Note that although each $R'_i$ is defined for all $n$-tuples over $D'$, it is known to hold for some of them (the $R'_1$-component of $R'_i$), and it’s known not to hold for others (the $R'_2$-component).

As a result, given a partial structure $A$, there are too many $A$-normal structures. We need then to provide constraints to restrict the acceptable extensions of $A$. In order to do that, a further auxiliary notion is introduced (see Mikenberg, da Costa, and Chuaqui 1986). A pragmatic structure is a partial structure to which a third component has been added: a set of accepted sentences $P$, which represents the accepted information about the structure’s domain. (Depending on the interpretation of science that is adopted, different kinds of sentences are introduced in $P$: Realists will typically include laws and theories, whereas empirists will add certain laws and observational statements about the domain in question.) A pragmatic structure is then a triple $A = (D, R_1, P)_{i \in I}$, where $D$ is a nonempty set, $(R_i)_{i \in I}$ is a family of partial relations defined over $D$, and $P$ is a set of accepted sentences. The idea is that $P$ introduces constraints on the ways that a partial structure can be extended (the sentences of $P$ hold in the $A$-normal extensions of the partial structure $A$).

We can now formulate the concept of quasi-truth. A sentence $\alpha$ is quasi-true in $A$ according to $B$ if (1) $A = (D, R_1, P)_{i \in I}$ is a pragmatic structure, (2) $B = (D', (R'_i)_{i \in I})$ is an $A$-normal structure, and (3) $\alpha$ is true in $B$ (in the Tarskian sense). If $\alpha$ is not quasi-true in $A$ according to $B$, we say that $\alpha$ is quasi-false (in $A$ according to $B$). Moreover, we say that a sentence $\alpha$ is quasi-true if there is a pragmatic structure $A$ and a corresponding $A$-normal structure $B$ such that $\alpha$ is true in $B$ (according to Tarski’s account). Otherwise, $\alpha$ is quasi-false.

The idea, intuitively speaking, is that a quasi-true sentence $\alpha$ describes not the whole domain to which it refers, but only an aspect of it – the one modeled by the relevant partial structure $A$. After all, there are several different ways in which $A$ can be extended to a full structure, and in some of these extensions $\alpha$ may not be true. As a result, the notion of quasi-truth is strictly weaker than truth: Although every true sentence is (trivially) quasi-true, a quasi-true sentence is not necessarily true (since it may be false in certain extensions of $A$).

To illustrate the use of these notions, let us consider a simple example. As is well known, Newtonian mechanics is appropriate to explain the behavior of bodies under certain conditions (say, bodies that, roughly speaking, have “low” velocity, are not subject to strong gravitational fields, etc.). But with the formulation of special relativity, we know that if these conditions are not satisfied, Newtonian mechanics is false. In this sense, these conditions specify a family of partial relations, which delimit the context in which the theory holds. Although Newtonian mechanics is not true (and we know under what conditions it is false), it is quasi-true; that is, it is true in a given context, determined by a pragmatic structure and a corresponding $A$-normal one (see da Costa and French 2003).

An important feature to note here is that a sentence and its negation can both be quasi-true. Of course, inconsistent sentences are not quasi-true in the same $A$-normal structure, but they can still both be quasi-true – as long as they are true in some $A$-normal structure. In other words, as defined above, if a theory is quasi-true, it is consistent (given that it is true in some full $A$-normal
structure). But in some contexts, we may need to assert that an inconsistent theory is quasi-true. How can we do that?

Here is a way. If a theory \( T \) is inconsistent, we say that \( T \) is quasi-true in a partial structure \( A \) if there are "strong" subsets of \( T \)'s theorems that are true in some \( A \)-normal structure. (We take "strong" to be a pragmatic notion, involving theories that are explanatory, have significant consequences, accommodate the relevant phenomena, etc.) In general, there are infinitely many "subtheories" of \( T \) that meet this condition. Of course, the interesting cases to consider are those in which \( A \) is a "good" pragmatic structure, in the sense that it reflects well the informal counterpart of \( T \).

For example, let \( T \) be naïve set theory, formulated in first-order logic. In this case, in the pragmatic structure \( A \), the set \( P \) of basic statements is constituted by statements that are typically taken to be unproblematic, such as the statement that asserts the existence of the union of two sets and the statement that expresses the comprehension schema restricted to a given set. In this case, there is only one relation in \( A \), the membership relation, which is taken to hold for certain pairs of sets. Hence, \( T \) is quasi-true in \( A \), given that there are several subtheories of \( T \) that are quasi-true, for instance, Zermelo-Fraenkel set theory, Quine’s NF and ML, and von Neumann-Bernays-Gödel set theory.

Additional examples of this sort can be provided, for instance, with the earlier formulations of the calculus. Such formulations, articulated in terms of infinitesimals, were inconsistent, but again they have "strong" consistent subtheories. Similarly, even though the conjunction of quantum mechanics and relativity theory is inconsistent, it can still be quasi-true, given the existence of strong consistent subtheories.

Of course, this construction presupposes, for the usual reasons, a meta-theory that is strong enough. Moreover, the construction is formulated in classical first-order logic, but it can be easily extended to higher-order logics, as in Frege’s system, or to other logics, using the theory of valuations (see da Costa 1997).

Two points should be emphasized here: (1) The fact that inconsistent theories can be quasi-true doesn’t entail that every sentence is quasi-true. After all, given a partial structure \( A \), there exist sentences that aren’t true in any \( A \)-normal structure. (2) The fact that inconsistent theories can be both quasi-true also doesn’t mean that everything follows from the partial structures framework. After all, the logic of quasi-truth is paraconsistent (see da Costa et al. 1998). And as was pointed out above, in a paraconsistent setting, it’s not the case that everything follows from an inconsistency. As a result, the partial structures approach provides the right sort of framework to examine issues regarding inconsistency in science. In terms of the approach, it’s possible to represent, without triviality, inconsistent theories as being quasi-true.

Having said that, we can now return to the main issue under consideration, and discuss how the partial structures approach allows us to consider this issue in a new way.

4. INCONSISTENCY, BELIEF CHANGE, AND PRAGMATISM

The framework above indicates one way in which it’s possible to make sense and pursue inconsistent belief systems without triviality. Such an outcome is far from idle, given the various benefits, discussed above, of taking seriously inconsistent belief systems. This outcome is also significant in a different way. Over the years, Levi has articulated a sophisticated and robust form of pragmatism (see, in particular, Levi 1991). And I think Levi’s pragmatism can benefit from incorporating a more robust treatment of inconsistent belief systems. I turn to this topic now.

There are several components in Levi’s pragmatism, and I cannot possibly do justice to all of them here. For our present purposes, though, I focus only on those components that bear on the issue of the aims of inquiry, since focusing on these components will be sufficient to illustrate my point.

According to Levi:

The aim [of inquiry] is to find the true complete story of the world — that is, the complete story that is also error-free. Granting that the conception of a complete story is relative to a conceptual framework or a language used to represent potential states of full belief in that framework, how are we to understand truth or freedom from error as an aim of inquiry? I emphasize that we are concerned with truth as an aim of inquiry focused on revising doxastic commitments.

(Levi 1991, p. 58)

Of course, as Levi insists, the true, error-free description of the world is achieved not in a single step, but basically through a process of expansion — where new information is added to a belief system (Levi 1991, pp. 71–116) — and contraction — where some information is excluded from the system (ibid., pp. 117–64). The crucial requirement of the proposal, however, is that each stage in the belief change process should avoid error (ibid., p. 161).

Given that error has to be avoided at each step, and given that contradictions are (in a classical setting) necessarily false, it’s not surprising that Levi insists on the need for contracting as soon as we face a contradiction. As he points out, “expansion into inconsistency will . . . incur a maximum risk of error and, for this reason, will be resisted” (ibid., p. 90).
However, as Levi also emphasizes, to find the true complete story of the world, we need more than simply avoid error; we also need to search for informative descriptions of the world. But in searching for the latter, we often end up incurring in error. After all, the more informative a theory is, the less likely it is that the theory is true. This means that, to be able to obtain new and more information, we may have to risk error. As Levi puts the point:

Even though inquirers should be concerned to avoid error, they also should be concerned to obtain new information of value, and such curiosity can justify risking error to obtain new information. (ibid., p. 160)

The idea of being justified to risk error in order to get new information is, of course, exactly right. Nevertheless, as the cases of Bohr and Frege mentioned above illustrate, sometimes the only way to obtain new information is by entertaining inconsistent theories. But Levi resists this move.

What should be prohibited is being prepared to add new information to one’s evolving doctrine when one is certain that it is false. No amount of new information can be worth the importation of certain error. However, this prohibition argues against expanding into contradiction by deliberate (= inferential = inductive) expansion; but it allows one to risk expanding into inconsistency via routine expansion, provided that the chances of importing contraction are sufficiently low. (ibid.)

The point is clear. To deliberately import inconsistent information is never allowed; inconsistent information can be added to a belief system only via routine expansion, and even then, only if the risk of importing a contradiction is low enough.

Suppose, however, that the only way of obtaining certain bits of information is by deliberately expanding into inconsistency. Bohr’s case discussed above illustrates this situation – as well as the earlier formulations of the calculus and the conjunction of quantum mechanics and relativity theory. To obtain the relevant information, in each of these cases, it looks as though one is forced to expand into inconsistency. Now, as long as the underlying logic is paraconsistent, there need not be anything unacceptable here. If we are searching for quasi-truth, these are all cases of inconsistent quasi-true theories. And by exploring the relevant partial structures, new information can be obtained, thus opening up the way for the formulation of better, consistent successor theories.

In other words, when faced with a contradiction, we need not embrace it, even in a paraconsistent setting. Given that we are not committed to the existence of true contradictions, eventually we will need to contract (or radically change) our inconsistent belief system. In this way, false information is not being permanently included in the system. The inconsistent information is included provisionally – the resulting system is taken only to be quasi-true – and all the information that depends on the inconsistency is tracked. And by exploring the resulting inconsistent but nontrivial system, we can make better informed decisions regarding exactly how, and when, to contract. In this way, the search for the true, error-free description of the world can be better implemented by allowing for a process of belief change that incorporates inconsistency.

5. CONCLUSION

Levi has developed an illuminating and systematic approach to belief change and the nature of inquiry. The considerations above suggest that, by making (more) room for inconsistency in his approach, he could achieve the pragmatist goals he has articulated so clearly in a more efficient way. Of course, this doesn’t mean that he would need to adopt the whole framework outlined here (in terms of partial truth and partial structures), although this framework provides one way in which it’s possible to combine inconsistency, informativeness, and avoidance of error in a systematic and unified form.

Perhaps all that is needed is just to adopt an underlying paraconsistent logic, given that this would allow one to explore inconsistent domains without triviality. As noted above, adopting such a logic doesn’t amount to endorsing the existence of true contradictions, and so false information will not be permanently added to our belief system. All that is required is to use paraconsistent logic as a mechanism of consequence generation. In this sense, the logic is simply an engine of inquiry, a further tool in the exploration of the world. By incorporating such a tool, Levi’s goals can be articulated still further, without any additional risk of error, and in a way that yields exactly the same results he obtains when we have a consistent domain (given that paraconsistent logic agrees with classical logic in consistent situations). In the end, this may not be a bad deal after all!

REFERENCES


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Levi on Risk

Nils-Eric Sahlin

In Plato’s Apology, Socrates declares:

Probably neither of us knows anything really worth knowing: but whereas this man imagines he knows, without really knowing, I, knowing nothing, do not even suppose I know. On this one point, at any rate, I appear to be a little wiser than he, because I do not even think I know things about which I know nothing.

First-order knowledge is important, but second-order knowledge of what one does or does not know is even more important: That is, it is essential to have what we might call “epistemic self-knowledge.” Scientists are knowledge-driven. That is why inductive methods are so popular in science. Knowledge is a good thing, but there are situations in which we require more – in which we require wisdom and thus epistemic self-knowledge. For example, when our theories fail to deliver results, when we are forced to find new theories or create new hypotheses, it is vital to know what one does not know.

Indirectly, Socrates tells us something about rational decision making. When everything is propitious, when we can represent our knowledge and our values with unique probability distributions and precise utility functions, we can simply maximize expected utility. We can use one of the classical theories – for example, Ramsey’s, or Savage’s, or Jeffrey’s theory. But when we are uncertain about the extent of our knowledge, when things are indeterminate, or when we are uncertain about our preferences, we know that the traditional theories will be of little or no use to us.

Isaac Levi saw this early, before others, and developed an alternative to the classical theories of rational decision making. Levi’s theory allows the agent to be uncertain (or indeterminate) in his probability assessments, and to have equivocal preferences. In this chapter I do not discuss the pros and cons of Levi’s decision theory. I have done that elsewhere. Instead, I want to focus on one of his less well-known papers: “A Brief Sermon on Assessing