1. INTRODUCTION

Is identity a fundamental concept? Of course, it all depends on what it takes for something to be fundamental. For the purposes of this paper, I will consider identity’s fundamentality as exhibiting four features, which, if satisfied, also provide four reasons to think that a fundamental concept is involved (see Bueno [2014]): (1) Identity is presupposed in every conceptual system; (2) it is required to characterize individuals; (3) it is not possible to define it, and (4) it is assumed for the intelligibility of quantification.

But what exactly is identity? Is it a property, a relation, or something else altogether? How is it related to indiscernibility? Is it a logical, a metaphysical, a pragmatic concept? These are all important issues, and there is much to be said, and much has been said, about them. But in the present article, I will try to remain, as much as possible, neutral on the various possible answers to these questions. (I intend to return to them in future work. For some discussion, see Williams [1989].)

In a recent paper, Décio Krause and Jonas Arenhart have challenged the very idea that identity is (or can be) fundamental, resisting each of the four reasons, listed above, that I offered in its support (Krause and Arenhart [2015]). In what follows, I will discuss the most significant of their complaints and provide a defense of the centrality—in fact, the fundamentality—of identity. Krause and Arenhart have raised a formidable challenge, and provided a variety of significant arguments for their case. Here I will only be able to focus on some of them. But I do plan to address the remaining ones in a companion paper.

2. IDENTITY, PROPERTIES AND CONCEPTUAL SYSTEMS

The first reason to consider identity to be fundamental emerges from the fact that it is presupposed in any conceptual system (Bueno [2014], p. 325). Concepts lump things together: those that fall under the same concept, in contrast to those that do not. To apply a concept, the identity of the objects need not be assumed, but the identity of the concept is required. Otherwise, it is not clear whether the objects under consideration fall under the extension or the anti-extension of the concept. It is crucial to determine that it is the same concept rather some other, different from it, which is being considered. Concepts are the same when they encode the same properties. (So we are dealing here with an intensional notion of concept. For reasons that I will return to in a moment, I do not think that an extensional understanding of concepts is minimally adequate.)

Vague concepts support this assessment. With regard to these concepts, there are clear cases in which they apply, clear cases in which they do not, and at least some cases in which it is not
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clearly determined whether they apply or not (the so-called *borderline cases*). The difficulty vague concepts raise emerges from their application conditions. Given the borderline cases, it is not always clearly determined what falls in the extension or in the anti-extension of a vague concept. But even vague concepts have identity conditions that are specified enough for them to be applied. One uses the clearly specified cases (those in which the concepts clearly applies and those in which it does not) to determine, within limits, the concept’s positive and negative extension, acknowledging that for a certain region of its domain, it may not be clearly determined which is which. Note, however, that the scope of the borderline cases (in which one finds lack of determinacy) is very constrained. It may not always be clear, say, whether something is red or orange (perhaps it is a light shade of the former, a dark shade of the latter?). But there is no doubt that the color in question is definitely *not* purple, blue or green. The clearly determined cases constrain significantly the scope of the borderline cases. The point still remains that, in the end, identity conditions for concepts, including vague ones, allow them to be applied. (For a selection of different approaches to vagueness, see Keefe and Smith (eds.) [1997].)

To resist the argument to the effect that identity is presupposed in concept application, Krause and Arenhart present it in the following terms:

> in order to determine the extension of a concept, we must determine also its complement. Things that fall under a concept cannot be also in the complement of the concept: those are *distinct* things. So, identity would be required in order to *distinguish* the items in the extension of a concept and the items belonging to its complement. To illustrate this point, let us assume that a concept C is given together with objects o₁ and o₂, so that o₁ falls under C and o₂ does not fall under C. In this case, o₁ is distinct from o₂. So, identity is required, given that it is identity that enables meaningful application of concepts (by allowing such a distinction between the extension and its complement). (Krause and Arenhart [2015], p. 2.)

In response, it is not clear to me that this a good formulation of the priority of identity in conceptual systems. First, it assumes an extensional conception of concepts. According to this view, a concept is identified with its extension, namely, those objects that fall under the concept. This conception, however, is clearly inadequate, since it lumps together intuitively distinct concepts: *creature with heart* and *creature with kidney* are distinct concepts, although presumably extensionally identical. Second, as formulated above, the argument presupposes the identity of objects, since the fact that object o₁ falls under concept C and object o₂ does not is taken to be enough to distinguish them. But as Krause and Arenhart correctly note ([2015], pp. 2-3), this begs the question against those who deny that some objects, such as quantum particles, have well-defined identity conditions (for discussion of this point, see French and Krause [2006]). Third, the issue here is *not* the identity of objects, but rather, as noted above, the fact that the application of concepts requires that concepts have identity conditions, so that things that fall under the *same* concept are lumped together. The proposed reconstruction of the argument fails to take this point into account.

But instead of invoking identity at all, Krause and Arenhart provide an alternative strategy to approach the issue, which invokes indiscernibility instead of identity in concept application. On their view:

> [We can] take indiscernibility as a primitive and recognize that it does not collapse on identity. We believe that the fact that indiscernibility can be analyzed without necessarily implying identity in some systems of logic shows that there is not a necessary equivalence between these notions. At the very least, it is logically possible that the relations of discernibility and difference are not the same, with discernibility being a weaker notion. In this case, there is an alternative way to understand the situation envisaged by Bueno without necessarily using

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*Identity in Physics and Elsewhere* by [Krause and Arenhart](http://example.com), 2015. (p. 2)
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identity. If this is correct, then identity is not really fundamental for the meaningful application of concepts. (Krause and Arenhart [2015], p. 3.)

This is a suggestive strategy, and one that makes good sense given the underlying conceptual framework that Krause, in particular, but also Arenhart, favor. This is a framework that allegedly takes indiscernibility, rather than identity, as basic in the development of a set theory in which not every object is assumed to have well-defined identity conditions: a quasi-set theory (see French and Krause [2006]).

There are, however, problems with this strategy. First, even if indiscernibility is taken as primitive, this does not entail that identity is not presupposed. In fact, the very attempt at distinguishing indiscernibility and identity presupposes identity! After all, as Krause and Arenhart note: “it is logically possible that the relations of discernibility and difference are not the same, with discernibility being a weaker notion” ([2015], p. 3; italics added). But this means that identity itself is presupposed so that the relations of discernibility and identity are not the same. We thus have a dilemma: If the two notions were the same, the attempt at distinguishing them would be moot. But if they are distinct, then as opposed to what Krause and Arenhart claim, identity is assumed in the end. Problems emerge in either case.

Second, there are indeed systems in which indiscernibility is primitive and does not entail identity (such as Schrödinger logics and quasi-set theory; see French and Krause [2006], Chapters 7 and 8). But these systems ultimately presuppose identity. Consider, for instance, the first three axioms of quasi-set theory, which express that the primitive concept of indiscernibility, ‘≡’, is an equivalence relation (French and Krause [2006], p. 277):

\[
\begin{align*}
(A_1) & \ \forall x \ (x \equiv x) \\
(A_2) & \ \forall x \ \forall y \ (x \equiv y \rightarrow y \equiv x) \\
(A_3) & \ \forall x \ \forall y \ \forall z \ ((x \equiv y \land y \equiv z) \rightarrow x \equiv z)
\end{align*}
\]

In each of these axioms, identity is ultimately presupposed. After all, the same variables need to be used in each side of ‘≡’ in (A₁), and the same variables need to be bounded by the quantifiers in each side of ‘→’ in (A₂) and (A₃). Thus, despite the weaker notion of indiscernibility, it is unclear that the alternative approach offered here does succeed in avoiding identity altogether. (I will consider below whether the fact that identity is presupposed in the metalanguage rather than in the object language can be easily dismissed, and will argue that it provides reason to believe that even at the object language identity is ultimately presupposed.)

3. IDENTITY, DISTINGUISHABILITY, INDIVIDUALITY

The second consideration that favors identity’s fundamentality is that identity is required to specify individuals (see Bueno [2014], pp. 326-328). There are two minimal (necessary, but not sufficient) conditions for something to be (considered) an individual:

(i) The first is distinguishability: individuals are distinguishable from one another and from other things (to use a neutral term that need not assume the identity of the things under consideration). This condition can be interpreted either ontologically or epistemologically. Ontologically, individuals just are the kind of things that have distinguishing features. Epistemologically, one can use these features to distinguish both one individual from one another and from other things as well.
(ii) The second condition is *re-identification*: individuals can be re-identified. Something can be identified as being the same individual, and as being different from others. This is typically an epistemological condition, but it can also be interpreted ontologically: the ontological features individuals have are precisely what allow such individuals to be re-identified.

These two conditions are very minimal (in either their epistemological or ontological readings), and the idea is not to defend them as ultimately characterizing individuals. They fail to provide sufficient conditions for that. But they are arguably necessary: it is difficult to see how something can be an individual without being distinguishable from other things and without being re-identifiable (as the same individual). For my purposes here, it is enough to note the significance that these minimal conditions be present: if they are violated, it is unclear that an individual is involved.

Note, however, that both conditions presupposed identity: (i) an individual can be distinguished from others that are different from it, and (ii) the same individual can be re-identified (singled out and uniquely determined). If identity were somehow inapplicable, none of these conditions could be satisfied, and as a result, individuality would fail.

Krause and Arenhart object:

One could accept that individuals are characterized by at least these two conditions and still hold that identity is not fundamental. In fact, for the conclusion that identity is fundamental to go through, one would still have to add the premise that every object is an individual, or some other claim to that effect. However, if there are objects that are not individuals (according to the above definition), then they do not obey the conditions for individuality (by definition of a non-individual), so that they may be characterized according to conditions that do *not* require identity (and then identity is not fundamental if that is really the case). (Krause and Arenhart [2015], p. 7.)

But this is not quite right. The claim is only that the characterization of individuals presupposes identity, not that every object is an individual. The latter claim is neither required nor true. It is, in fact, false: drops of water are not individuals. Under normal conditions, drops of water cannot be distinguished from one another nor can a drop be water be re-identified as the same over time. Despite that, there’s no doubt that drops of water exist!

It may be argued that to identify water drops we can just tag the atoms that characterize them. In this way, water drops can be distinguished and re-identified by distinguishing and re-identifying the relevant atoms that compose them. This suggestion, however, does not go very far. For whether atoms themselves can be tagged is a contentious issue in this debate. After all, the process of tagging presupposes that the objects that are being tagged can be distinguished from one another (that is, they differ from other objects), so that different objects are never receive the same tag. However, clearly this presupposes identity, something that those who deny that identity can be applied to quantum particles would not grant.

Note that in stating that something (e.g. a water drop) is not an individual, the two minimal conditions above were invoked. It had to be established that the relevant non-individual cannot be distinguished from others, nor can one re-identify them as the same over time. Since identity is presupposed in the formulation of both conditions, identity is still presupposed in the denial that these conditions apply to certain things (such as water drops). Thus, identity is still fundamental.

### 4. Identity’s Indefinability

The third argument in favor of identity’s fundamentality is that identity cannot be defined, even in languages that have the technical resources to implement such a definition (see Bueno [2014], pp. 328-329, and references therein). Consider Leibniz laws, formulated in second-order logic:
∀x ∀y (x = y ↔ ∀P (Px ↔ Py)),

here ‘P’ ranges over suitable properties. (A careful analysis of Leibniz’s principle of the identity of indiscernibles in the context of Leibniz’s philosophical system is developed in Rodriguez-Pereira [2014].) Clearly, identity is presupposed in the statement of this law: the same variables need to be used on both sides of all bi-conditionals, otherwise the statement will fail to express the intended content. Obviously, objects a and b are not the same if objects c and d, distinct from them, turn out to have the same properties, whatever properties a and b may have in the end. Alternatively, to avoid referring to a plurality, an object is not the same if some other object has the properties that it has.

Moreover, even in languages that cannot fully express identity (e.g. propositional ones), identity is presupposed. Consider:

(A v ¬A).

In order to formulate a logical law, the same variable A needs to be used on both sides of the disjunction. Otherwise, the intended content will not be captured. As a result, identity is, once again, assumed.

Krause and Arenhart complain:

The fact that we use identity in elaborating our conceptual scheme does not force upon us the identity of the objects we are dealing with, and this is the point to be emphasized. This, we think, answers Bueno’s related claims concerning propositional logic. In fact, in the language of classical propositional logic, the occurrences of A in a tautology like (A or not A) are occurrences of the same variable, but we could simply say that they are two occurrences of the variable A without mentioning identity at all, just by emphasizing the number (as we [did] before, by distinguishing the various tokens of a type). Anyway, this use of identity is in another level than the one which questions its applicability to a certain realm. Indeed, this notion does not matter for the possible consideration of a metaphysics involving objects like quantum non-individuals. (Krause and Arenhart [2015], p. 14.)

Krause and Arenhart are right in noting that the presupposition of identity in the metalanguage does not require the identity of the objects referred to in the object language. These are indeed separate issues. But the point is that the very formulation of identity, in fact the very formulation of logical laws, requires identity. Otherwise, the right content fails to be expressed: two objects are not the same if they have different properties; ‘A or not-B’ is not a logical law unless A and B are the same variables. Identity, of course, is presupposed throughout.

Note that, in the case of (A or not-A), it is not enough to claim that we have “two occurrences of the variable A without mentioning identity at all. [we] just [emphasize] the number (as we [did] before, by distinguishing the various tokens of a type)” (Krause and Arenhart [2015], p. 14). After all, for the formula (A or not-A) to be properly expressed, we would need to assume that we have two occurrences of variables of the same type. But, once again, identity is still being assumed.

Krause and Arenhart ([2015], p. 14) will insist that: “this use of identity [in the metalanguage] is in another level than the one which questions its applicability to a certain realm. Indeed, this notion [in the metalanguage] does not matter for the possible consideration of a metaphysics involving objects like quantum non-individuals”. But it does matter. After all, the same point applies when non-individuals are formulated in quasi-set theory. As we saw, the axioms of quasi-
set theory, which state that the indistinguishability relation of the theory is an equivalence relation, presuppose identity. Identity is presupposed even by a theory that attempts to do without it. In this sense, identity is indeed fundamental.

I do not deny that quasi-set theory posits objects (namely, non-individuals) for which identity alleged is not defined (or does not apply). The point is that the very formulation of the theory requires identity, and in this respect identity is fundamental.

5. QUANTIFICATION AND IDENTITY

The fourth argument for identity’s fundamentality is that the intelligibility of quantification presupposes identity (Bueno [2014], p. 329). In classical logic, if each object in the domain of quantification is $F$, then every object in the domain is $F$. That is the reason why we can infer that $\forall x Fx$ from $Fa$ if ‘$a$’ is arbitrary, that is, if it is a parameter rather than a proper name. Nonetheless, this is the case only if each distinct object in the domain is in the range of the universal quantifier. After all if the same object were repeatedly quantified over, and if in each case it were $F$, there would be no reason to conclude that every object is $F$.

Krause and Arenhart disagree. On their view:

One could apply a proof-theoretic kind of semantics in which the meaning of the quantifiers is fixed by the axioms and rules we use for such logical constants, such as the standard ones in first-order or in higher-order logics, and nothing about the domain is said from this purely formal point of view. According to this approach, the way quantifiers work is determined by the axioms we use, and not by the intended interpretation we have for them on a Tarski-style semantics. So, the universal quantifier in particular gets its meaning independently of identity. (Krause and Arenhart [2015], p. 15.)

Of course, there is nothing wrong with a purely formal approach to quantification, and to logical consequence more generally. But suppose we are interested in understanding what it means to quantify over certain things (to use a neutral term that does not presuppose identity), rather than just to operate with the quantification symbol. In this case, a purely formal approach will not be very helpful, since the question we are interested in cannot even be asked: quantification over something presupposes that something is being quantified over, rather than certain symbols are just being manipulated. (Note that no existence claim is made here, since the quantifiers can all be ontologically neutral; see Azzouni [2004], Bueno [2005], and Bueno [2013].)

But an additional objection needs to be considered in this context:

Even in classical semantics, one can have an alternative interpretation that goes without mentioning each object of the domain: it is related to […] generalized quantifiers. In a nutshell, call $|F|$ the class of objects of the domain that have $F$, and let $D$ be the domain of […] interpretation. The interpretation for $\forall x Fx$ can now be stated simply as saying that $D$ is a subset of $|F|$. For instance, we may say that $|F|$ is the class of all (just two) Oxygen atoms in a molecule of $O_2$ with no need of identifying them. In the same vein, the interpretation for $\exists x Fx$ means that $|F|$ is not empty. For instance, we may say that in [a] Helium atom in the fundamental state, there exists one electron with spin UP in a given direction, with no need to identify it (in fact, this is impossible according to standard quantum mechanics). In neither of the mentioned cases [is] the identity of the objects […] quantified over required. (Krause and Arenhart [2015], p. 16.)

However, this will not do. This way of formulating generalized quantifiers presupposes set theory. After all, the interpretation of $\forall x Fx$ is cashed out in terms of the fact that the domain of interpretation $D$ is a subset of $|F|$, that is, the class of objects of the domain that have the property
F. Similarly, the interpretation of $\exists x \, F x$ is cashed out in terms of the fact that the class $|F|$ is not empty. Now, the usual formulations of set theory, in which class and subset are characterized, require the extensionality axiom:

$$x = y \iff \forall z \, (z \in x \leftrightarrow z \in y).$$

But this axiom, in turn, presupposes identity. In fact, it provides identity conditions for sets. So, this version of generalized quantifiers is unable to avoid the commitment to identity in the end.

Perhaps generalized quantifiers could be formulated in quasi-set theory rather than in a standard set theory? This is certainly a possibility, which may seem to avoid the concern. But the suggestion does not go very far. After all, as we saw, the axioms of quasi-set theory themselves presuppose identity, and so we are still left with it. Moreover, if we were to invoke quasi-set theory at this point, we would end up in a circle. The semantics of the quantifiers is given in a theory (quasi-set theory) that is required to interpret those quantifiers. However, the semantics was supposed to give us that interpretation, but it can only be formulated in a theory (quasi-set theory) that invokes precisely those quantifiers that need to be interpreted in the first place. The difficulty, thus, still stands.

Krause and Arenhart may question why a classical formulation of the quantifiers is being assumed, and may insist that it would be less contentious in the debate to invoke some other understanding. In response, the classical formulation of quantifiers has the advantage of being well understood, and if a new formulation of quantification is advanced, it is crucial to articulate the details of this formulation in a way that is intelligible and provides a clear understanding of the quantifiers in question, ideally without being parasitic on the classical quantifiers. Until this is done, a proper understanding of the quantifiers in quasi-set theory has not been provided yet.

6. CONCLUSION

For the reasons discussed above, I still think that identity is fundamental, in the sense that: (1) it is presupposed in every conceptual system; (2) it is required to characterize individuals; (3) it cannot be defined, and (4) it is needed for the intelligibility of quantification.

Note that the fundamentality of identity need not be understood in terms of conditions of possibility of the application of concepts, or the characterization of individuals, or quantification. Fundamentality is not understood here as a metaphysically substantive notion (this is part of a broad defense of an empiricist view, after all). As I noted in the beginning of this paper, I also do not try to settle here what kind of thing identity is: a relation, a concept, or something else altogether. Identity’s fundamentality does not require deciding this sort of metaphysical issue.

But one particular moral seems to emerge: it does not matter how hard we try, we just cannot get rid of identity. Perhaps because it is so fundamental in the end?

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