Intuitive Knowledge[1]

Elijah Chudnoff

Abstract: In this paper I assume that we have some intuitive knowledge—i.e. beliefs that amount to knowledge because they are based on intuitions. The question I take up is this: given that some intuition makes a belief based on it amount to knowledge, in virtue of what does it do so? We can ask a similar question about perception. That is: given that some perception makes a belief based on it amount to knowledge, in virtue of what does it do so? A natural idea about perception is that a perception makes a belief amount to knowledge in part by making you sensorily aware of the concrete objects it is about. The analogous idea about intuition is that an intuition makes a belief amount to knowledge in part by making you *intellectually* aware of the *abstract* objects it is about. I expand both ideas into fuller accounts of perceptual and intuitive knowledge, explain the main challenge to this sort of account of intuitive knowledge (i.e. the challenge of making sense of intellectual awareness), and develop a response to it.

In the *Rules*, Descartes insists that "we can best learn how mental intuition is to be employed by comparing it with ordinary vision," and throughout his writings he characterizes intuition by drawing analogies between it and perception.[2] There are various such analogies one might draw: one might, for example, compare intuitive and perceptual phenomenology, or intuitive and perceptual justification.[3] In this paper I compare intuitive and perceptual knowledge.

By a piece of intuitive knowledge I mean a belief that amounts to knowledge because it is based on an intuition. Suppose, for example, you believe that circles are

[1] I presented earlier versions of this paper at a meeting of the Arché Methodology Workshop at St. Andrews, and a conference associated with the Descartes Lectures at the Tilburg Center for Logic and Philosophy of Science. The Q&A sessions on both occasions were instructive, and I thank those in attendance for their participation. I thank John Bengson, Ed Erwin, Uriah Kriegel, Kirk Ludwig, Susanna Siegel, Ernie Sosa, and Anand Vaidya for additional helpful comments and discussion.

[2] The quote is from Rule 9, (Descartes 1985), pg 33.

[3] Recent discussions of such comparisons include (Bealer 1998), (Bengson 2010), (Bonjour 1998), (Chudnoff 2011a, 2011b), (Huemer 2001), (Parsons 1980, 2007), (Sosa 2007, 2009), and (Tieszen 1989, 2005).

symmetrical about their diameters on the basis of your intuition that circles are symmetrical about their diameters. And suppose your belief amounts to knowledge because it is based on your intuition. Then this belief is a piece of intuitive knowledge: you have intuitive knowledge that circles are symmetrical about their diameters.

I assume that we have some intuitive knowledge. The question I want to take up is about its explanation. If your intuition that circles are symmetrical about their diameters makes your corresponding belief amount to knowledge, then there must be some features of your intuition in virtue of which it does so. My question is about these features. What are they? That is: given that some intuition makes a belief based on it amount to knowledge, in virtue of what does it do so?

We can ask a similar question about perception: given that some perception makes a belief based on it amount to knowledge, in virtue of what does it do so? I discuss this question more fully below, but a plausible initial idea is this. If a perception makes a belief based on it amount to knowledge, it does so at least partly by making you sensorily aware of the items in your environment that this belief is about. Suppose the explanation of intuitive knowledge is analogous to the explanation of perceptual knowledge. Then something similar should be true for intuition: if an intuition makes a belief based on it amount to knowledge, it does so at least partly by making you intellectually aware of the abstract objects that this belief is about.[4]

[4] Many deny the existence of abstract objects. And some deny that intuitions are about abstract objects; see, for example, (Goldman 2007). The reasons for these denials are often epistemological. In this paper I assume that there are abstract

In my view, any attempt to draw an analogy between the explanation of intuitive knowledge and the explanation of perceptual knowledge is bound to invoke intellectual awareness of abstract objects at some point.[5] Many philosophers would take this to be a reason to avoid drawing such an analogy. But I am more optimistic, and the bulk of this paper is dedicated to explaining why.

In section 1, I begin to flesh out the analogy by articulating a view of perceptual knowledge. In section 2, I explain what the analogous view of intuitive knowledge is and make clear the main challenge to it that I intend to address. The main challenge is to make sense of intellectual awareness of abstract objects. In sections 3, 4 and 5, I develop a response to that challenge.

1. Perceptual Knowledge

There is a difference between having a perceptual experience and being sensorily aware of an object. Contrast the following two reports:

(1) Smith has a perceptual experience representing that the traffic lights are green.

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objects and that our intuitions are, at least largely, about them. These assumptions are appropriate since one of my aims is to show that even given them, we can still develop a reasonable explanation of intuitive knowledge.

[5] I am interested in exploring an analogy between what it is in virtue of which some intuitions make beliefs based on them amount to knowledge and what it is in virtue of which some perceptions make beliefs based on them amount to knowledge. Sometimes I will use the expression "the analogy between intuitive and perceptual knowledge," where this might suggest I am picking out surface similarities, such as the fact that both are non-inferential. Throughout, however, I am concerned with explanation.

(2) Smith is sensorily aware of, e.g. sees, green traffic lights.

These report on different states. (1) could be true of Smith even if (2) is not: Smith might be hallucinating. (2) could be true of Smith even if (1) is not: though Smith might see green traffic lights, he might not be able to make out that they are green, for they might be too far away, or they might be pointing in the opposite direction.

Though having a perceptual experience and being sensorily aware of an object are distinct, in a *perception* they are importantly related.[6] Brian O'Shaughnessy puts it this way:

> Whenever a person perceives-that p, he both believes that p and perceives something which is relevant to p's truth-value. For example, one believes that the traffic lights are green, and sees the greenness of the traffic lights.[7]

I disagree with some of the details of O'Shaughnessy's formulation, but the main idea seems correct to me. If Smith perceives that the traffic lights are green, he both has a perceptual experience representing that the traffic lights are green and is sensorily aware of green traffic lights, and perhaps, as O'Shaughnessy suggests, the greenness of the traffic lights.

My preferred general formulation of the idea is this:

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[6] Perception is a success state: perceiving is veridical and non-hallucinatory.
[7] (O'Shaughnessy 2002), pg 319.

> Whenever you perceive that p, there is some q (maybe = p) such that you have a perceptual experience representing that q and you are sensorily aware of an item o that makes q true.

Let me highlight two differences between this formulation and O'Shaughnessy's. First, I have replaced the loose relation of being relevant to p's truth-value with the tighter relation of making p true. This difference motivates the second. Suppose Smith checks his speedometer and sees that he is driving at 60 m.p.h. In this case he perceives that p (= that he is driving at 60 m.p.h.) and he has a perceptual experience representing that p, but he is not aware of a truth-maker for p. Consider, however, the proposition that q (= that his speedometer reads 60 m.p.h.). In perceiving that p, Smith also has a perceptual experience representing that q, and he is aware of a truth-maker for q—e.g. the position of his speedometer's needle.

O'Shaughnessy's formulation does not distinguish between p and q: Smith is aware of something relevant to the truth-value of both. But while he is only aware of something that is evidence for p, he is aware of something that makes q true. Perhaps his perception puts him in a position to know both that p and that q. In the case of p, however, it seems that his knowledge also depends on his background beliefs about speedometers. So this knowledge might not wholly derive from his perception. In the case of q, on the other hand, a case can be made for thinking that Smith's knowledge does wholly derive from his perception—even if it can be defeated by certain background beliefs. If you are skeptical about q itself, then just consider an even more basic proposition about the colors and shapes of things. My

point is that every perception, at some level, both represents propositions about the world and makes us sensorily aware of the chunks of the world that make those propositions true. When I talk about perceptual knowledge I have in mind knowledge of propositions at this level, i.e., knowledge that wholly derives from a perception.

The explanatory thesis that I will advance is about perceptual knowledge so understood. Here it is:

> Perceptual Knowledge (PK): If a perception makes a belief that p based on it amount to knowledge, it does so in virtue of 1) being an experience in which it perceptually appears to you that p, and 2) being an experience in which you are sensorily aware of an item o, such that 3) o makes p true.

Similar views can be found throughout the literature on perception and perceptual knowledge.[8] The view that comes closest is Mark Johnston's:

> Sensory awareness discloses the truthmakers of our immediate perceptual judgments. Those truthmakers are external spatio-temporal particulars, which sensory awareness makes available for immediate demonstration. The structural elements (objects, stuff, their qualities, and the relations in which they stand) in those truthmakers are then recombined in immediate

[8] (Russell 1992, 1997) and (Husserl 1950) are early proponents of similar views. More recent proponents of similar views include (McDowell 1982) and (Fumerton 2006).

> judgment…if I am seeing a spoon on the table, and judge accordingly, then I typically know that there is a spoon on the table….I typically know these things because the judgments in question are reliably formed from their respective truthmakers, which awareness makes manifest.[9]

The main difference between Johnston's view and my own is that his view leaves out perceptual experiences. On Johnston's view, Smith sees the greenness of the traffic lights and judges that the traffic lights are green. On my view, Smith sees the greenness of the traffic lights, has a perceptual experience representing that the traffic lights are green, and judges that the traffic lights are green. This is a potentially significant difference between our overall views of perception, but it is not something that requires discussion here.

(PK) is *prima facie* plausible.[10] There are, however, three potential obstacles to it, which I would like to highlight.

The first obstacle is that the three main notions it relies on require clarification. These are perceptual content (e.g. its perceptually appearing that p, or one's having a perceptual experience representing that p), sensory awareness (e.g. seeing o), and truth-making.

The second obstacle is that these three main notions must be aligned properly in perception. Specifically, the sorts of things of which we can be sensorily aware must be the sorts of things that can be truth-makers for the sorts of

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[9] (Johnston 2006), pages 282 and 289.
[10] I discuss additional positive evidence one might produce in favor of (PK) in (Chudnoff ms1).

propositions that we perceptually represent. What sorts of things might these be? Johnston includes objects, stuff, their qualities, and the relations in which they stand. O'Shaughnessy includes tropes such as the greenness of the traffic lights. We might also include events, states of affairs, and facts.

The third obstacle is that the "in virtue of" claim that (PK) includes must not imply a sufficiency claim, lest it run afoul of Gettier counter-examples. That is, it must be possible for a perception to make a belief that p based on it amount to knowledge *in virtue of* conditions (1), (2), and (3), even though the fact that a perception meets conditions (1), (2), and (3) does not *entail* that it can make a belief that p based on it amount to knowledge. Consider fake barn cases.[11] If Henry spots a barn in fake barn county, then he has a perceptual experience representing that there is a barn, and he sees an item—the barn, or a barn-hood trope—such that it makes true the proposition that there is a barn. But Henry is not in a position to know that there is a barn.

I believe that all three obstacles can be met. I cannot defend that belief here, so in this paper I must leave it as an assumption. I will add, however, that the obstacles are neither uniformly challenging, nor uniformly damaging if not overcome. The third might be the most challenging but it is the least damaging, since if we cannot meet it, then we can just take conditions (1), (2), and (3) to be the central necessary conditions in an account of perceptual knowledge and note that

[11] Cf. (Goldman 1976). Some philosophers deny that fake barn cases are Gettier cases. For a recent discussion see (Lycan 2006). Whether they are or not does not affect the views of perceptual and intuitive knowledge I discuss here.

further work is needed to deal with Gettier cases.[12] The first and second obstacles are very damaging if they cannot be overcome. They would undermine the foundations of the account of perpetual knowledge. The first obstacle is not overall that worrisome, however, since it is likely not very challenging to overcome.[13] The second obstacle is more worrisome. One might worry in particular that we can only be sensorily aware of non-propositionally structured items, such as objects and stuff, but that it is only propositionally structured items, such as states of affairs and facts, that can be truth-makers. One case that weakens the worry is that of events: events can be seen and can be truth-makers. I suspect that further reflection will show that there are other ontological categories with instances that can be both objects of sensory awareness and truth-makers, but this must remain a promissory note for now.[14]

2. Analogy and Challenge

If intuitive knowledge is analogous to perceptual knowledge, then the following should hold:

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[12] I defend the view that P might obtain in virtue of Q even though Q does not entail P in (Chudnoff ms2). I focus on the significance this view has for thinking about Gettier cases and the nature of perceptual knowledge in (Chudnoff ms1).

[13] The literatures on sensory awareness, perceptual content, and truth-making are individually large and collectively vast. A sample of sympathetic works might include: (Dretske 1969), (Johnston 2006) on sensory awareness; (Siegel 2005, 2011) on perceptual content; (Armstrong 2004), (Mulligan et al 1984), and (Rodriguez-Pereyra 2005) on truth-making.

[14] (Mulligan et al 1984) and (Johnston 2006) are the only discussions I am aware of that directly address this issue. They come down in favor of the view that we can be sensorily aware of truth-makers.

> Intuitive Knowledge (IK): If an intuition makes a belief that p based on it amount to knowledge, it does so in virtue of 1) being an experience in which it intuitively appears to you that p, and 2) being an experience in which you are intellectually aware of an item o, such that 3) o makes p true.

The structural parallels between (IK) and (PK) should be obvious. The differences are that intuitive appearance replaces perceptual appearance and intellectual awareness replaces sensory awareness.

What motivation is there for exploring (IK)? One motivation is phenomenological. Descartes' practice of characterizing intuition by analogy with perception is motivated by experienced similarities between the two. Consider the following two claims:

(1) If $a < 1$, then $2 - 2a > 0$

(2) $\sqrt{7} + \sqrt{10} > \sqrt{3} + \sqrt{17}$

If you are like me, then after a moment (1) should seem true and (2) should remain opaque. It is possible to calculate that (2) is true. But for most of us, its truth never becomes intuitively apparent in the way that the truth of (1) becomes intuitively apparent. Reflecting on the experience I have when (1) intuitively appears to me to be true, I find that I also seem to be aware of what makes it true: I can "see" how a's

being < 1 makes 2a smaller than 2, and so 2 – 2a > 0. I do not want to dwell here on the aptness of this phenomenological description.[15] But if you do find it apt, then it is natural to wonder whether the similarities between perception and intuition run deeper than phenomenology. For example, in addition to similarities between intuitive and perceptual phenomenology, maybe there are similarities between intuitive and perceptual knowledge.

This seems to me motivation enough to explore whether a view like (IK) can be made to work. In order to best argue for (IK) one would have to work out its details and compare it to alternative accounts of intuitive knowledge. What I want to do here, however, is explore whether there is any reason to think we can make it that far. Is there any reason to think that (IK) can be made to work at all?

The main challenge to (IK) derives from the fact that the objects of intellectual awareness, if there are any, are abstract. This matters because abstract objects are causally inert. And this matters because according to our best understanding of the best-understood form of awareness, i.e. sensory awareness, awareness requires causal dependence:

> The thought of my fleeting perception as a *perception* of a continuously and independently existing thing implicitly contains the thought that if the thing had not been there, I should not even have *seemed* to perceive it. It really should be obvious that with the distinction between independently existing objects and perceptual awareness of objects we already have the general

[15] I have discussed this issue at length in (Chudnoff 2011).

> notion of causal dependence of the latter on the former, even if this is not a matter to which we give much reflective attention in our pre-theoretical days.[16]

Here Strawson gives an informal gloss on the causal theory of perception, or more accurately, the causal theory of sensory awareness. William Child gives a sharper formulation for the case of seeing:

> (I) If S sees o then:
>
> (a) There is a state of affairs reportable by a sentence of the form 'It looks to S as if …,' and
>
> (b) o is causally responsible for this state of affairs.
>
> (II) Conditions (a) and (b) are requirements by our ordinary concept of vision.[17]

Child's formulation can be generalized by replacing 'If S sees o' with 'If S is sensorily aware of o' and condition (a) with a condition that attributes a perceptual experience to S without committing to its modality. If condition (II) seems too strong it can be weakened: (a) and (b) might hold as a matter of *a posteriori* necessity, or natural law.[18] The causal theory of sensory awareness need not favor

[16] (Strawson 1979), pgs 103 – 104 in the reprint in (Dancy 1988).
[17] (Child 1996), pg 141.
[18] Strawson and Child think the necessity is conceptual. I remain neutral.

any particular theory of the nature of perceptual experience: intentionalists, sense-data theorists, adverbialists, and disjunctivists can all embrace it.[19]

The causal theory of sensory awareness, as I take it here, only purports to identify a (conceptual, *a posteriori*, or natural) necessary condition on sensory awareness. This is enough to generate a problem for (IK). One might aim to *refute* (IK) by arguing that the casual theory of sensory awareness can be generalized into a causal theory of any kind of awareness, which then rules out the possibility of awareness of abstract objects. But there is little motivation for the generalization. A more defensible ambition is to raise an explanatory challenge to (IK). If we didn't have the causal theory, we wouldn't understand the nature of sensory awareness. The gap in our knowledge would be too large. Suppose there is such a thing as intellectual awareness. Then without a theory that plays a role analogous to the role that the casual theory plays in giving us an understanding of sensory awareness, we wouldn't understand the nature of intellectual awareness. Again, the gap in our knowledge would be too large. But we do in fact lack such a theory. So we do not understand the nature of intellectual awareness. Further, it is unclear how to go about rectifying this situation. So the rational thing to do is to suspend deployment of the notion of intellectual awareness, including its deployment in (IK), until we better understand its nature.

This challenge to (IK) is modest. It highlights a task proponents of (IK) must complete in order for their endorsement of (IK) to be rational, but it does not aim to demonstrate that this task is impossible, or otherwise refute (IK). Committed

[19] The case of disjunctivism is the trickiest. Child seems to me to make a good case for thinking that the causal theory and disjunctivism are compatible, however.

opponents of (IK) might be disappointed. But this modesty is a virtue, since the challenge gets off the ground without relying on strong premises about knowledge and reference.[20]

3. Explanation and Awareness

In the last section I rejected the idea of generalizing the causal theory of sensory awareness into a causal theory of every kind of awareness. There is, however, an alternative way of generalizing the causal theory, which I do find plausible. Building on Child's formulation of the causal theory of seeing, we might formulate the generalization this way:

(I) If S is aware of o then:
  (a) There is a state of affairs reportable by a sentence of the form 'It appears to S as if …,' and
  (b) o (or some fact about o) is part of the explanation of this state of affairs.

(II) Conditions (a) and (b) are conceptually (or *a posteriori*, or naturally) necessary.

"If S is aware of o" replaces "If S sees o"; "'It appears to S as if…'" replaces "'It looks to S as if…'"; "o (or some fact about o) is part of the explanation of" replaces "o is

[20] Cf. (Benacerraf 1973), (Field 1989).

causally responsible for"; and I have made explicit the possibility that (a) and (b) might hold with *a posteriori* or natural necessity, rather than conceptual necessity. We might call the resulting theory the *becausal theory of awareness*, since "because" is a general explanatory connective, covering both causal and non-causal explanation.

The becausal theory of awareness restricted to sensory awareness is as plausible as the causal theory of sensory awareness. The reason why is that whenever a causal relation obtains so does a becausal relation. So any consideration in favor of the necessity of a causal relation is a consideration in favor of the necessity of a becausal relation. And any consideration against the necessity of a becausal relation is a consideration against the necessity of a causal relation.

The becausal theory of awareness remains plausible when we lift the restriction to sensory awareness. Consider, for example, the awareness we have of events in our past. Plausibly, I count as aware of some event in my past only if that past event is part of the explanation of my current recollective experience.[21] Consider also the awareness we have of our current phenomenal states. Take a pain. Plausibly, I count as aware of this pain only if it is part of the explanation of the experience in which I am aware of it. On some views the pain and the experience in which I am aware of it are ontologically inseparable, so that it is mistaken to think that the pain *causes* my awareness of it.[22] Still, a becausal relation might obtain. In this case it is a non-causal becausal relation.

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[21] (Martin and Deutcher 1966). One might reject the causal view and still accept a becausal view. See the lectures on memory in (Malcolm 1975).
[22] For discussion of these issues see the papers in (Kriegel and Williford 2006).

And this brings us to the relevance of the becausal theory of awareness to our present inquiry. Even if the supposed objects of intellectual awareness cannot cause intuitive appearances, they might be parts of non-causal explanations of intuitive appearances. We might endorse the following non-causal theory of intellectual awareness—which is just another restricted version of the becausal theory of awareness:

(I) If S is intellectually aware of o then:

(a) There is a state of affairs reportable by a sentence of the form 'It intuitively appears to S as if …,' and

(b) o bears some non-causal responsibility for this state of affairs.

(II) Conditions (a) and (b) are conceptually (or *a posteriori*, or naturally) necessary.

I have been working up to the idea that there exists a theoretical option proponents of (IK) might pursue in meeting the challenge to provide us with some understanding of intellectual awareness. The theoretical option is to endorse a non-causal theory of intellectual awareness. The mere existence of this option might give one encouragement, but it falls short of addressing the challenge. To address the challenge, the proponent of (IK) must show that the conditions of the non-causal theory of intellectual awareness can be met, that *it is possible for an abstract object to bear some non-causal responsibility for an intuitive appearance*.

To show this, the proponent of (IK) must show two things:

First, he must show that intuitive appearances can be targets of non-causal explanations. This involves considering various types of non-causal explanation, considering the nature of intuitive appearances, and exploring whether some of them admit of some type of non-causal explanation.

Second, he must show that abstract objects can play roles in non-causal explanations of intuitive appearances. Might circularity, for example, play a role in non-causally explaining the mental state I am in when it intuitively appears to me that circles are symmetrical about their diameters?

4. Non-Causal Explanations of Intuitive Appearances

There are three parts to this section. In part a, I introduce a few different non-causal explanations, and identify the kind that I believe applies to intuitive appearances. In part b, I give reasons to think that intuitive appearances admit of this kind of non-causal explanation. In part c, I explain why abstract objects can play roles in such explanations.

a. Non-Causal Explanation

Consider the following explanations:

- Xantippe became a widow because Socrates died.[23]

[23] Cf. (Kim 1974), (Ruben 1992), pg 223.

- Ice is water because it is composed of H2O.[24]
- This gas has temperature t because its constituent molecules have mean kinetic energy m.[25]
- My car is touching the pavement because its front wheel is touching the pavement.[26]
- My car is parked illegally because it is parked next to a fire hydrant.[27]
- My car is red because it is vermilion.
- Her hat doesn't hold its form because it is a bundle of straw tied together with string.[28]
- These items compose a bicycle because they are so arranged to allow locomotion on two wheels by peddling.
- These items compose *this* bicycle because they are so arranged to allow locomotion by peddling on *it*.
- This peddle is part of this bicycle because it is in part by peddling it that you can ride this bicycle.

These are non-causal explanations. Socrates' death, for example, did not *cause* Xantippe to become a widow, though she did become a widow *because of*, or *in virtue of*, his death.

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[24] Cf. (Achinstein 1985) pgs 228 - 237, (Ruben 1992), pg 218.
[25] Cf. (Achinstein 1985) pgs 228 - 237, (Ruben 1992), pg 219.
[26] Cf. (Owen 1992), pgs 71 - 81.
[27] Cf. (Owen 1992), pgs 71 – 81.
[28] Cf. (Ruben 1992), 221.

My interest here is in the last three. These are explanations of why some items compose something of a certain kind, of why some items compose some particular, and of why one item is part of some particular. They seem to me to be perfectly good explanations. They are, however, potentially controversial. The reason why is that acceptance of them is easily confused with commitment to something else—answers to what Van Inwagen has called the Special and General Composition Questions.[29]

Let us say that an analysis of F is a non-circular, finitely statable, set of necessary and sufficient conditions for F's application. Then the two questions can be formulated as follows:

> Special Composition Question: what is the analysis of *there is something that the xs compose*?
>
> General Composition Question: what is the analysis of *the xs compose y*?

Any answer to one of these questions is controversial, so I want to make clear my commitments with respect to them.

The only view with respect to these questions that I am committed to is that Nihilism, and positions that approach it are false. Nihilists think that *there is something that the xs compose* never applies. Positions approach Nihilism insofar as they assert that *there is something that the xs compose* applies much less frequently

[29] (Inwagen 1990).

than we ordinarily think it does. So a view on which no items ever compose a bicycle approaches Nihilism. Nihilism and its kin are unpopular, and I will assume they are false.

What I am more concerned to show is that I am not committed to any specific non-Nihilist answers to the Special and General Composition Questions. Nor am I committed to the view that they have answers.[30] The reason why is that the Special and General Composition Questions are distinct from the following two question *schemas*:

> Special Explanatory Question Schema: given that there is something that the Φs compose, in virtue of what is there something that the Φs compose?
>
> General Explanatory Question Schema: given that the Φs compose φ, in virtue of what do the Φs compose φ?

These are question schemas, not questions, since 'Φs' and 'φ' are schematic letters, not variables. When you replace the schematic letters with terms, then you get a question. For example:

> Given that these items [imagine me demonstrating some items] compose something, specifically a bicycle, in virtue of what do they do so?

[30] (Markosian 1998) argues that they do not.

That is a question. And I believe that it has an answer, namely that the items in question are so arranged to allow locomotion on two wheels by peddling. This is a particular explanation of a particular fact. Accepting it does not commit one to a general answer to a general question about the analysis of *there is something that the xs compose*.

Let us call the following combination of theses Compositional Particularism:

(A) At least some instances of the Special and General Explanatory Question Schemas have answers.

(B) The Special and General Composition Questions do not have answers.

It is not clear whether Compositional Particularism is true since it is not clear whether (B) is true. But it should be clear that Compositional Particularism is a coherent theoretical position.[31] In this paper I will be a practicing Compositional Particularist. I will discuss particular explanations of particular instances of part-hood and composition, without committing to general views about the analysis of part-hood and composition.

b. Intuitive Appearances

[31] The obvious inspiration is moral particularism. See, for example, (Dancy 2004).

Let us call the explanations I have been discussing mereological explanations. I believe that intuitive appearances admit of mereological explanations. If this is so, then they must have parts.

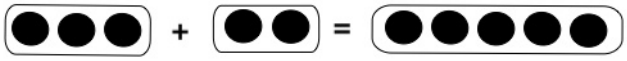
On one view, intuitive appearances either do not have parts, or have only two parts. On this view an intuitive appearance is wholly individuated by its content and what attitude you take toward its content.[32] So the intuitive appearance that circles are symmetrical about their diameters is wholly individuated by its being an intuitive—rather than, e.g., a perceptual—appearance and its representing that circles are symmetrical about their diameters—rather than, e.g., that squares are symmetrical about their diagonals. The attitude and content of an intuitive appearance are likely properties of it, rather than parts. But if they are parts, then, on this view, they are the only two parts.

There is an alternative view of intuitive appearances on which they have more significant mereological structure. According to Husserl, our intuitions are "founded on" our thoughts, perceptions, and imaginings, where being founded on is

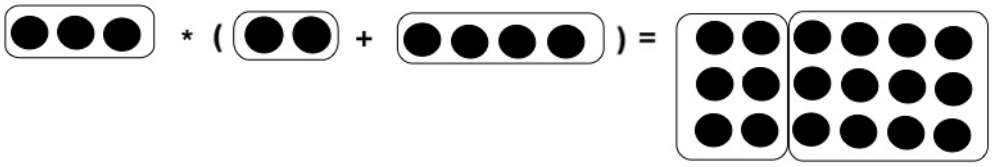
[32] There is disagreement about the nature of this attitude. Some think that it is a doxastic attitude or disposition, such as belief or having an inclination to believe; (Williamson 2008). Others think it is a *sui generis* attitude, such as having something intellectually seem to be the case; (Bealer 1998), (Huemer 2001).

a mereological notion.[33] Consider some examples.

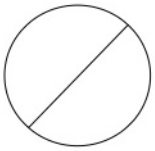
(1) 3 + 2 = 5.



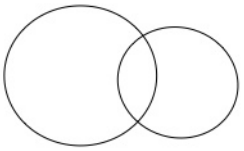
(2) a(b + c) = ab + ac.

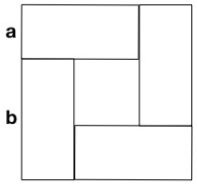


(3) Circles are symmetrical about their diameters.



(4) Two circles can intersect in at most two points.





(5) $(a + b)^2 \geq 4ab$.
(6) $(a + b)^2 = 4ab$ just when $a = b$.

(7) If $a < 1$, then $2 - 2a > 0$.

Visual imagery dominates reflection on (1) through (6). In (3), (4), and (6) especially the imagery is dynamic: it involves manipulating figures in your mind's eye. There is more than just visual imagery, however. In (2) through (6) you have to "see" the general in the particular. In (7), there is no visual imagery, just a train of thought.

[33] See (Husserl 2001), especially pgs 281 – 304. The exact analysis of Husserl's notion of foundation is controversial. See (Smith et al 1982), (Fine 1995), (Correia 2004) for discussion. Husserl's view that intuitions are founded on perceptions, imaginations, and thoughts was standard among those in the phenomenological tradition. See, for example, (Lévinas1995). More recently, (Parsons 1980, 2007) and (Tieszen 1989, 2005) have developed views of intuition that draw on Husserl's.

These examples suggest that there is an intimate connection between reflections—thoughts, perceptions, imaginings—and intuitive appearances. Intuitive appearances "arise out of" reflections. The Husserlian view is that some of your thoughts, perceptions, and imaginings are parts of the intuitive appearances that you experience. On this view intuitive appearances are not additional experiences that occur alongside of thoughts, perceptions, and imaginings in your stream of consciousness. Rather, they are constituted by collections of thoughts, perceptions, and imaginings.[34]

Why believe this view?

Intuitive appearances are experiences and should be individuated as finely as their phenomenology. It is possible for there to be two intuitive appearances that p, which differ phenomenally. Consider (6). It might intuitively appear true to you as you imaginatively manipulate the four inner rectangles in the accompanying figure so that they approach squares. Or it might intuitively appear true to you because in supposing that a = b you think of $(a + b)^2 = 4ab$ as $(a + a)^2 = 4aa$, i.e. as $(2a)^2 = 2^2a^2$, which is obvious. There is a phenomenal difference between these two intuitive appearances. That suggests they include more than their attitude and content. The view that they include some thoughts, perceptions, and imaginings tells us what more. And it does so in a way that accounts for our phenomenological observations, since, on the face of it, the difference between the two intuitive appearances consists

[34] In this paper I focus on part-hood, a many-one relation elements in a plurality—some thoughts, perceptions, and imaginings—bear to a unity—an intuitive appearance. Constitution is a relation between two unities—a *sum of* thoughts, perceptions, and imaginings and an intuitive appearance. I focus on this relation elsewhere; (Chudnoff 2011b).

in the fact that the first includes manipulating figures and the second includes manipulating symbols.[35]

One opposing view is that reflections merely cause intuitive appearances. If intuitions are partly individuated by reflections, however, then the relation between the two cannot be merely causal. Proponents of the causal view need to rule out the claim about individuation.

One way to rule out the claim about individuation is to argue that reflections are merely phenomenally co-present with intuitive appearances. The idea is that while there is a difference in your overall phenomenology when you reflect on (6) in one or another way, there is no phenomenal difference in the intuitive appearances themselves. It is not clear what might motivate such a view. It is *prima facie* implausible. Suppose you have a pain in your knee while it intuitively appears to you that (6) is true. Your pain is merely phenomenally co-present with your intuitive appearance. But surely your reflections on (6) bear a more intimate phenomenal relation to your intuitive appearance than the pain does. It is the way your reflections line up or fall into place that gives your intuitive appearance its forcefulness, its quality of putting forward a proposition as one you ought to believe.

An alternative way to bolster the causal view, or at least undermine the Husserlian view, is to argue that what I have been characterizing are not intuitions,

[35] The argument in this paragraph is similar to arguments in favor of the view that perceptual experiences have mental paint, qualia, or sensational properties; see, e.g., (Block 1996), (Peacocke 1983). In those arguments two perceptual experiences representing that p are supposed to differ phenomenally. My argument, however, is compatible with the view that phenomenology supervenes on intentional content, since in the case I am considering it can be the intentional contents of the thoughts, perceptions, and imaginings that make the phenomenal difference between the two intuitive appearances.

but inferences. The idea is that you infer (1) through (7) from premises provided by your reflections. Inferences, as opposed to intuitions one might think, should be individuated in part by the considerations that lead to them.

It is not clear how to *prove* that one has intuited rather than inferred something. One might appeal to phenomenology. Consider (2). Does the transition from reflection on the figure to "seeing" that (2) is true *feel* like an inference? I doubt it. One might argue that there are sub-personal inference-like processes going on, which introspection does not reveal. But the aim is to classify experiences, not the psychological mechanisms that underlie them. And here introspection does seem relevant. One might also appeal to epistemological considerations in criticizing the inferential view. If the transition from reflection on the figure to "seeing" that (2) is true is an inference, then it is a bad inference, since it is an inference to a claim about an infinite number of cases from a premise about one case. But surely your episode of "seeing" justifies you in believing (2), and so is not just a bad inference.[36]

The Husserlian view that intuitive appearances have thoughts, perceptions, and imaginings as parts is a viable alternative to the view that intuitive appearances are wholly individuated by attitude and content.[37] I have been considering

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[36] The standard proof of (2) uses mathematical induction. Nothing like that is going on when you "see" that (2) is true. One might wonder whether there is a less secure but still reasonable sort of inference that occurs, such as an inference to the best explanation. This is implausible. In "seeing" that (2) is true, do you really compare it to alternative explanations of the data represented by the figure? No. But if not, then this is a bad inference to the best explanation.

[37] There are various arguments one might press against it. One might argue, for example, that intuitive appearances often *outlast* the reflections that give rise to them. Or one might argue that some intuitive appearances are *immediate* and do not involve reflection. I do not have the space here to respond to these and other arguments against the Husserlian view; but see (Chudnoff 2011a, 2011b).

phenomenological evidence in its favor. Now I want to explore some of its attractions as a theoretical framework for thinking about intellectual awareness and intuitive knowledge.

c. The Explanatory Role of Abstract Objects

The foregoing sub-sections suggest that intuitive appearances admit of a kind of non-causal explanation, specifically mereological explanation. Suppose it intuitively appears to you that p. Call this intuitive appearance A. And suppose some experience E, for example your experience of manipulating a figure in your mind's eye, is part of A. Why is E part of A? In many cases a partial answer of the following form can be given:

> (Expl) E is part of A partly because: 1) E presents x; 2) A presents y; and 3) x bears R to y.

The idea is that experience E is part of intuitive appearance A partly because there is a relation R between their intentional objects—i.e. what E presents and what A presents.

Explanations conforming to (Expl) can be given for experiences other than intuitive appearances. Suppose Smith has a temporally extended auditory experience of a melody, ABC etc. He hears note A, then note B, then note C, etc. His auditory impression of note A is part of his experience of the melody, as are his

auditory impressions of notes B and C. Why? At least partly because his auditory impression of note A presents a playing of note A; his experience of the melody ABC etc presents a playing of the melody ABC etc; and the playing of note A is part of the playing of the melody ABC etc.

Consider now an explanation conforming to (Expl) for an intuitive appearance. Suppose it intuitively appears to Smith that (1) is true. His visual representation of the accompanying figure is part of this intuitive appearance. Why? Plausibly:

> (ExplA) Smith's visual representation is part of Smith's intuitive appearance partly because: 1) His visual representation presents an instance of the operation of taking the union of two sets of dots; 2) his intuitive appearance presents the operation of adding two numbers (by presenting a fact about it); and 3) the operation of taking the union of two sets of dots is isomorphic to the operation of adding two numbers, i.e., there is an isomorphism between the structure (N, +) and the structure (Set of sets of dots, ∪).[38]

Here the relation R between the intentional object of the part-experience, i.e. Smith's visual representation, and the whole-experience, i.e. Smith's intuitive appearance, is

[38] One might worry that clause (2) is problematic because it imports the assumption that Smith's intuitive appearance makes him aware of addition. But (ExplA) is a partial explanation of the fact that Smith's visual representation is part of his intuitive appearance, not an argument that his intuitive appearance makes him aware of addition.

not itself part-hood. It is rather the relation of *x being an instance of an operation isomorphic to y*.

If (ExplA) is true, it is explanatory. It gives us information about Smith's visual representation that enables us to understand why it is part of Smith's intuitive appearance. Suppose Smith has a concurrent temporally extended visual experience of a dance. Why is Smith's visual representation of the figure *not* part of his experience of the dance? Through some quirk in his brain there might be a causal relation between them. Through some crazy background belief system there might be an inferential relation between them. But there cannot be a part-hood relation between them. His visual representation of the figure is not the right sort of experience to be part of his experience of the dance, though it is the right sort of experience to be part of his intuitive appearance. (ExplA) informs us about the features that make this so.

If (ExplA) is true, then an abstract object, specifically the operation of adding two numbers, bears some non-causal responsibility for Smith's intuitive appearance. Consider, first, a general bit of reasoning about explanations conforming to (Expl). Suppose E is part of A partly because E presents x; A presents y; and x bears R to y. Plausibly, a whole exists partly because its parts compose it. So we have: E exists partly because E presents x; A presents y; and x bears R to y. It trivially follows that E exists partly because x bears R to y. So, E exists partly because y is a certain way, namely such that x bears R to it. If E's existence partly depends on y's being a certain way, then y bears some responsibility for E's existence. So we have: y bears some responsibility for E's existence. The sort of

explanation we have been considering is mereological, so non-casual. Hence: y bears some non-causal responsibility for E's existence. This general bit of reasoning remains cogent when instantiated. Considering the instantiation relevant to (ExplA), then, we should conclude that the operation of adding two numbers bears some non-causal responsibility for Smith's intuitive appearance.

Here is a more speculative, supplementary consideration. Suppose we allow that counterfactuals with impossible antecedents can make substantive claims. This is a controversial view, but adopting it for certain purposes can have heuristic value.[39] And suppose, further, that (ExplA) is true. Then it seems reasonable to conclude that had the operation of adding two numbers not been isomorphic to the operation of taking the union of two sets of dots, then Smith would not have had his intuitive appearance.[40] So Smith's intuitive appearance counterfactually depends on the properties of adding two numbers. And that lends further support to the view that the operation of adding two numbers bears some non-causal responsibility for Smith's intuitive appearance.

One might object that it is the properties of Smith's visual representation that explain why it is part of Smith's intuitive appearance, not the properties of addition. But according to (ExplA) it is a relation between the two that matters, and so the properties of both that matter.

One might object that Smith would experience his intuitive appearance even if his visual representation did not have the property of presenting an instance of an

[39] It is gaining acceptance; see, for example, (Nolan 1997), (Brogaard and Salerno 2007), (Kment 2006a, 2006b), (Salerno and Brogaard 2007).
[40] I am also assuming that Smith's intuitive appearance is not over-determined.

operation isomorphic to addition. But this is not so *if* (ExplA) is true.[41] On the supposition that (ExplA) is true, Smith's intuitive appearance depends on his visual representation having the property of presenting an instance of an operation isomorphic to addition. This shows that (ExplA) is not trivial: its truth makes some substantive demands on the nature of Smith's intuitive appearance.

The non-triviality of (ExplA) is worth noting. It is not the case that every intuitive appearance will have some explanation conforming to (Expl). This is important because just as some perceptual experiences, e.g. hallucinations, fail to make us aware of the subject matter they represent, so some intuitive appearances fail to make us aware of the subject matter they represent. One way for this to happen is for such an intuitive appearance to lack an explanation conforming to (Expl).

The foregoing supports concluding this much: if (ExplA) is true, then addition bears some non-causal responsibility for Smith's intuitive appearance. On the face of it, (ExplA) could be true. It is not logically impossible. And it does not make any demands on abstract objects inconsistent with their nature. Absent a reason to think otherwise, we should think that it is possible for (ExplA) to be true. Suppose this is so. Then it is possible for addition to bear some non-casual responsibility for Smith's intuitive appearance. And so it is possible for the conditions of the non-causal theory of intellectual awareness to be met: that is, it is possible for an abstract object to bear some non-causal responsibility for an intuitive appearance.

[41] Once again, the assumption of no over-determination is in place.

5. Sensory and Intellectual Awareness

(IK), recall, is the following view of intuitive knowledge:

> Intuitive Knowledge (IK): If an intuition makes a belief that p based on it amount to knowledge, it does so in virtue of 1) being an experience in which it intuitively appears to you that p, and 2) being an experience in which you are intellectually aware of an item o, such that 3) o makes p true.

The challenge to this view that I set out to address is to fill out our understanding of intellectual awareness enough so that we can get on with developing (IK) and comparing it with its competitors. Toward meeting this challenge, I've set out two main ideas. First, the idea that you are intellectually aware of an abstract object o only if o bears some non-causal responsibility for an intuitive appearance that you experience. Second, the idea that the way for o to bear this non-causal responsibility is for its characteristics to be part of the mereological explanation of why some other experiences—thoughts, perceptions, and imaginings—compose your intuitive appearance, and so why you experience the intuitive appearance that you do.

These ideas seem to me to provide us with some understanding of intellectual awareness. I will conclude by pointing out areas where that understanding could be improved.

First, though (Expl) provides a template for generating non-causal explanations of intuitive appearances, it is incomplete. The reason why is that it

does not specify what relations can play the role of R. As (ExplA) illustrates, the relation *x is an instance of an operation isomorphic to y* is one. But it is not the only one. Other plausible candidates are: *x is an instance of an operation homomorphic to y*, and *x is an instance of y*. Your visualization of two intersecting circles, for example, might be part of your intuitive appearance that (4)—the proposition that two circles can intersect in at most two points—is true partly because it presents an instance of two intersecting circles.

There is an analogous incompleteness in our understanding of how the conditions of the casual theory of sensory awareness might be met. It is not enough for an object o to be part of just any causal chain leading up to a perceptual experience. The causal chain must not be *deviant*. For example, an object o does not meet the necessary condition identified by the causal theory if a mad scientist's observations of o prompt him to stimulate your brain so as to generate random perceptual experiences. So just as we need to say what relations can play the role of R, we need to say what casual chains are appropriate.

Second, the non-causal theory of intellectual awareness does not give us sufficient conditions for intellectual awareness. The same can be said of the causal theory of sensory awareness. These are potential sources of incompleteness in our understanding of both intellectual and sensory awareness. But there is a way they might not be.

It could be that sensory and intellectual awareness are primitive in the way that Williamson thinks knowledge is primitive.[42] Truth is a necessary condition on

[42] See (Williamson 2002).

knowledge: if S knows that p, then p is true. But if Williamson is right, there is no way of expanding this condition into an analysis of knowledge, which would give us not only necessary but also sufficient conditions for knowledge. Sensory and intellectual awareness might be similar. That is, the becausal theories might identify necessary conditions on each type of awareness, though these cannot be expanded into analyses of sensory and intellectual awareness, which would give us not only necessary but sufficient conditions for them.

A third way our understanding of intellectual awareness might be improved has particular relevance to (IK). If (IK) is true, then the sorts of things of which we can be intellectually aware must be the sorts of things that can be truth-makers for the sorts of propositions that we intuitively represent. Nothing that I have said about intellectual awareness guarantees that this will be so. Once again, however, we are in a similar position with respect to sensory awareness and perceptual knowledge. In section 1, where I first discussed this sort of worry, I suggested that events at least are plausible candidates to be objects of sensory awareness and truth-makers for perceptually represented propositions. There are no abstract events. But there are various functions, operations, and mappings. In my view these are plausible candidates to be objects of intellectual awareness and truth-makers for intuitively represented propositions. This is an underexplored issue.

Fourth, finally, and perhaps most saliently: I have confined by discussion to mathematical examples, but, it seems, we have intuitive knowledge about, and intellectual awareness of, non-mathematical matters as well, such as freedom, beauty, rationality, justice, etc. Insofar as such intuitive knowledge derives wholly

from intuition, and not from background conceptual knowledge, I believe that (IK) applies to it as well.[43] Further exploration of this issue must be set aside for another occasion.

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[43] The notion of knowledge that derives wholly from intuition parallels that of knowledge that derives wholly from perception; see the discussion of this in section 1. For discussion of the role of background conceptual knowledge in intuitive knowledge broadly construed see (Ludwig 2007).

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